1. Suppose you are given an array of numbers, some of which are marked as icky, and you want to compute the length of the longest increasing subsequence of $A$ that includes at most $k$ icky numbers. Your input consists of the integer $k$, the number array $A[1..n]$, and another boolean array $Icky[1..n]$.

For example, suppose your input consists of the integer $k = 2$ and the following array (with icky numbers are indicated by stars):

$$\begin{array}{cccccccccccccccc}
3 & | & 1^* & | & 4 & | & 1^* & | & 5^* & | & 9 & | & 2^* & | & 6 & | & 5 & | & 3 & | & 5 & | & 9 & | & 7 & | & 9^* & | & 3 & | & 2 & | & 3 & | & 8^* & | & 4 & | & 6^* & | & 2 & | & 6^*
\end{array}$$

Then your algorithm should return the integer 5, which is the length of the increasing subsequence $4, 5^*, 6, 7, 9^*$.

(a) Describe an algorithm for this problem using dynamic programming.
(b) Describe an algorithm for this problem by reducing it to a standard graph problem.

Think about later:

2. Let $G$ be a directed acyclic graph whose vertices have labels from some fixed alphabet. Any directed path in $G$ has a label, which is a string obtained by concatenating the labels of its vertices. Recall that a palindrome is a string that is equal to its reversal.

Describe and analyze an algorithm to find the length of the longest palindrome that is the label of a path in $G$. For example, given the dag below, your algorithm should return the integer 6, which is the length of the palindrome $HANNAH$.

(a) Describe an algorithm for this problem using dynamic programming.
(b) Describe an algorithm for this problem by reducing it to a standard graph problem.