Deterministic Finite Automata (DFA)

Intuitively, a machine that uses constant amount of memory & reads input in pass (from left to right).

Exo. All strings over \{0,1\} with odd # of 0's

Example: All strings ending with 01

States: $q_0$: None of above
$q_1$: Just seen 0
$q_2$: Just seen 01

$\delta(q_0, 0)$ = $q_1$
$\delta(q_0, 1)$ = $q_0$

$\delta(q_1, 0)$ = $q_0$
$\delta(q_1, 1)$ = $q_2$

$\delta(q_2, 0)$ = $q_2$
$\delta(q_2, 1)$ = $q_0$

$\delta(q_0, 001) = \delta(\delta(q_0, 0), 01) = \delta(q_1, 01) = \delta(q_1, 0) = \delta(q_0, 0) = q_0$

$\delta(q_2, 01) = \delta(q_2, 0) = q_2$

Formal Def'n. A DFA is specified by 5 things:

- Initial state
- Set of states
- Input alphabet
- Transition function
- Set of accepting states
A DFA is specified by 5 things:

\[ M = (Q, \Sigma, s, \delta, A) \]

where

- \( Q \) is a finite set of states
- \( \Sigma \) is finite alphabet
- \( s \in Q \) is the start state
- \( A \subseteq Q \) is the set of accepting states
- \( \delta: Q \times \Sigma \rightarrow Q \) is the transition function

**Def**

Given transition function \( \delta \), define its extended transition function \( \delta^* : Q \times \Sigma^* \rightarrow Q \) inductively:

1. \( \delta^*(q, \varepsilon) = q \)
2. \( \delta^*(q, x) = \delta(\delta(q, x), y) \) if \( x = ay \) with \( a \in \Sigma, y \in \Sigma^* \)

**Def**

\( M \) accepts \( x \) iff \( \delta^*(s, x) \in A \)

Define \( L(M) = \{ x \in \Sigma^* : M \text{ accepts } x \} \)

**Exs**

- \( \Sigma = \{0, 1\} \)
  - a) all strings beginning with 001

```plaintext
\begin{center}
\begin{tikzpicture}
  \node (q0) at (0, 0) {$q_0$};
  \node (q1) at (1, 1) {$q_1$};
  \node (q2) at (2, 2) {$q_2$};
  \node (q3) at (3, 3) {$q_3$};
  \node (q4) at (1, -1) {$q_{\text{not}}$};

  \draw[->] (q0) -- (q1) node [midway, above] {$0$};
  \draw[->] (q0) -- (q4) node [midway, left] {$0$};
  \draw[->] (q1) -- (q2) node [midway, above] {$0$};
  \draw[->] (q1) -- (q4) node [midway, left] {$1$};
  \draw[->] (q2) -- (q3) node [midway, above] {$1$};
  \draw[->] (q4) -- (q2) node [midway, above] {$0$};

  \node at (4, 0) {$\vdash 0, 1$};
\end{tikzpicture}
\end{center}
```
b) all strings containing 001 as a substring

- \( q_0 \): none of above
- \( q_1 \): just seen 0 but not in \( q_2, q_3 \)
- \( q_2 \): just seen 00 but not found 001 yet
- \( q_3 \): found 001

\( A = \{ q_0, q_1, q_2 \} \)

generates to other pattern strings
pattern matching algm with \( O(n) \) time

c) all strings not containing 001

d) all strings with even \# of 0's and even \# of 1's
e) Strings with length divisible by 5