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GPS1 & HW1 available

Last Time:

strings

languages

ops on languages:

- union, intersection, complement
- concatenation

  $L_1L_2 = \{xy : x \in L_1, y \in L_2\}$
- Kleene star

  $$L^* = \bigcup_{i=0}^{\infty} L^i$$

Regular Languages

all langs obtainable from union, concat, star (starting from trivial base cases)

Formal Def'n by induction

(i) ϕ, $\{\varepsilon\}$, $\{a\}$ are regular langs $\forall a \in \Sigma$

(ii) if L_1, L_2 are regular langs, then so are $L_1 \cup L_2$, L_1L_2, and L_1^*

(only langs obtained by finite # appl'ns of (i), (ii) are regular)

Ex $\Sigma = \{0,1\}$

a) $\{1001\}$ is regular

$= \{1\} \cdot \{0\} \cdot \{0\} \cdot \{1\}$

b) $\{1001, 10, 1\}$ is regular

$= \{1\} \cdot \{0\} \cdot \{0\} \cdot \{1\} \cup \{1\} \cdot \{0\} \cup \{1\}$

c) all finite langs are regular.

d) $\{x \in \{0,1\}^* : |x| \text{ is } \underline{\text{odd}}\}$ is regular

$$= \{00, 01, 10, 11\}^* \cdot \{0,1\}$$
$$= \left(\left(\{0\} \cup \{1\}\right) \cdot \left(\{0\} \cup \{1\}\right)\right)^* \cdot \{0,1\}$$

⋮

Notation regular expressions

(i) ϕ, ε, a are regular expr for $\phi, \{\varepsilon\}, \{a\}$ $\forall a \in \Sigma$

(ii) if r_1, r_2 are regular exprs for L_1, L_2 resp,

then $(r_1 + r_2)$ is reg expr for $L_1 \cup L_2$

$(r_1 r_2)$ " " $L_1 L_2$

(r_1^*) " " L_1^*

Let $L(r)$ denote lang corresponding to expr r.

Ex (d) $\left((0+1)(0+1)\right)^* (0+1)$

Rmk:
- omit unnecessary parentheses
  - precedence order: $*, \cdot, +$
- shorthand: $r^+ = r \cdot r^*$ (one or more occurrence)
- a lang may have many diff. reg expr

  e.g. $(0+1) \cdot \left(\underbrace{(0+1)(0+1)}\right)^*$

  $(0+1)\left(00 + 01 + 10 + 11\right)^*$

Ex $(\Sigma = \{0,1\})$

a) all strings with 00 as a substring

$$\Rightarrow \quad (0+1)^* \, 00 \, (0+1)^*$$

b) all strings with 00 as a substring
and having even length

2 cases: (i) part before 00: even / part after 00: even

(ii) part before 00: odd / part after 00: odd

even / odd

$$\boxed{((0+1)(0+1))^* \, 00 \, ((0+1)(0+1))^* + ((0+1)(0+1))^*(0+1) \, 00 \, ((0+1)(0+1))^*(0+1)}$$

c) all strings with even # of 0's

$$\cancel{(00)^*}$$

... 0 ... 0 ... 0 ... 0 ...

$$(1^* 0 1^* 0 1^*)^* + 1^*$$

or $(1^* 0 1^* 0)^* 1^*$

or $1^* (0 1^* 0 1^*)^*$

$(1^* 0 1^* 0 1^*)^*$

d) all strings not beginning with 00

$$\cancel{00(0+1)^*}$$

cases: begin with 1 ✓
or begin with 01 ✓

boundary case: $\varepsilon, 0$

or begin ...

$$1(0+1)^* + 01(0+1)^* + \varepsilon + 0$$

$$= (1+01)(0+1)^* + \varepsilon + 0$$

e) all strings not containing 00 as a substring

$$\boxed{(1+01)^* (\varepsilon+0)} \quad \boxed{(1+01)^* + (1+01)^* 0}$$

$$= (1^* + 01^*)^* (\varepsilon+0)$$

11 1 1 1 0 1 1 1 1 1 0 ... $= (\varepsilon+0)(1^+0)^* 1^*$

f) all strings with even # of 0's and even # of 1's

idea - divide into blocks of two

$\cancel{00}$ 01 10 $\cancel{11}$

such blocks must be even

00 10 11 01 01 10

(# of 01 & 10 blocks) is even.

$$\boxed{\left((00+11)^* (01+10)(00+11)^* (01+10)(00+11)^*\right)^* + (00+11)^*}$$

or $\left((00+11)^* (01+10)(00+11)^* (01+10)\right)^* (00+11)^*$

L^*: $\boxed{\in L}\boxed{\in L}\boxed{\in L}\boxed{\in L}$

g) all strings with even # 0's
and # 1's div by 5
and not containing 01101

hard but possible!

h) $\{0^i 1^i : i \geq 0\}$

$\cancel{0^* 1^*}$

→ impossible!! (how to prove it?)

Some identities

$$L((r_1 + r_2)\cdot r_3) = L(r_1 r_3 + r_2 r_3)$$

$$L((r^*)^*) = L(r^*)$$

$$L((rs)^* r) = L(r(sr)^*)$$

$$L((r+s)^*) = L((r^* + s^*)^*)$$

$$= L((r^* s^*)^*)$$

don't memorize!

⋮