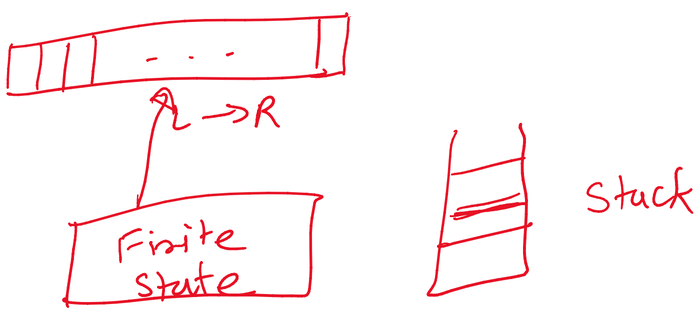
Tuesday, February 15, 2022 10:51 AM

Last Time CFL
(Intuition: DFA + Stack)

Non-deta Push down Automata (PDA)



- Closure Properties

Thm: $L_1 \xrightarrow{} S_1$, $L_2 \xrightarrow{} S_2$ are CFL
Then so are $L_1 \cup L_2$, $L_1 \cdot L_2$, L_1^*

$L_1 \cup L_2$: $S \to S_1 \mid S_2$

$L_1 \cdot L_2$: $S \to S_1 S_2$

L_1^*: $S \to S S_1 \mid \varepsilon$

Thm: If L_1 is CFL, L_2 is a regular lang.
Then $\underline{L_1 \cap L_2}$ is CFL
(Product construction of a PDA & a DFA)

Rmk: Not closed under intersection

Ex: $L_1 = \{0^i 1^i 2^k \mid i, k \geq 0\}$ CFL

$L_2 = \{0^i 1^k 2^k \mid i, k \geq 0\}$ CFL

$L_1 \cap L_2 = \{0^i 1^i 2^i \mid i \geq 0\}$ is not a CFL.

Not closed under complements.

Turing Machine (TM)

(early 1900) (how to define "all" that can be computed)

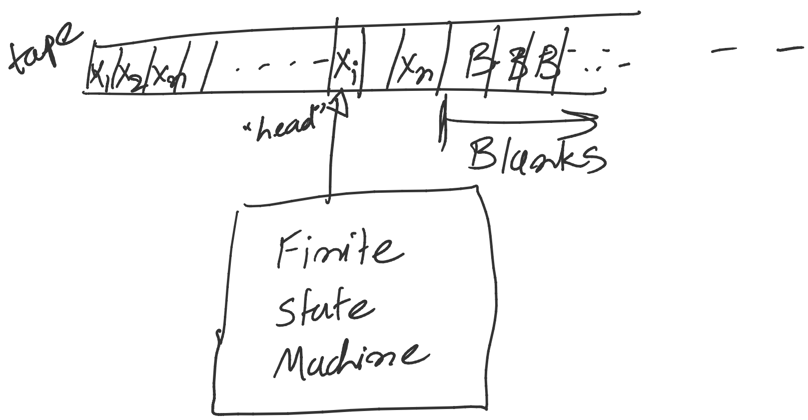
Turing (1936) - one of the simplest models.
Resembles modern computer.

Main features

- infinite memory
- can read & write both
- can go back & forth in the memory.

Implementation

- Unbounded tape
- read/write one symbol at current position ("head")
- Move head one position left/Right.



★ **Formal Def:** A deterministic TM is specified as

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, q_{acc}, q_{rej})$$

where

Q is a finite set of states

Σ finite alphabets for input

Γ finite "tape" alphabets $\quad \Sigma \subset \Gamma$

q_0 is start state

finite

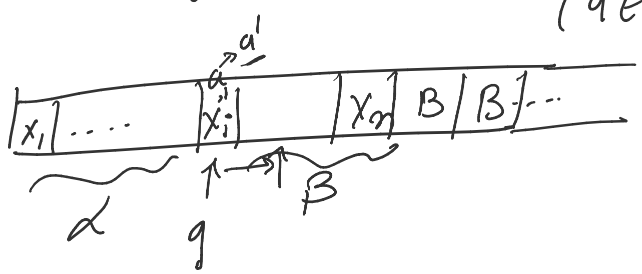
$q_0 \in Q$ is start state

$B \in \Gamma \setminus \Sigma$ is "blank" symbol

stop/halt states $\begin{cases} q_{acc} \in Q \text{ is a unique accept state.} \\ q_{rej} \in Q \text{ is a unique reject state} \end{cases}$

$$\delta : \underset{\substack{\uparrow \\ \text{current state}}}{Q} \times \underset{\substack{\uparrow \\ \text{read symbol}}}{\Gamma} \to \underset{\substack{\uparrow \\ \text{next state}}}{\underline{Q}} \times \underset{\substack{\uparrow \\ \text{write symbol}}}{\Gamma} \times \underset{\substack{\uparrow \\ \text{move left/right/stay}}}{\{L, R, S\}}$$

Def: A configuration (instanteneous identification (ID)) is a string $x_1 x_2 \dots x_{i-1}\, q\, x_i \dots x_n$ ($q \in Q$, $x_1 \dots x_n \in \Gamma$)



Def: (Single Move) Let $q \neq \underline{q_{acc}}, \underline{q_{rej}}$

If $\delta(\underline{q}, \underline{a}) = (q', a', \underline{S})$

$$\alpha q a \beta \underset{M}{\rightsquigarrow} \alpha q' a' \beta \qquad \alpha, \beta \in \Gamma^*$$

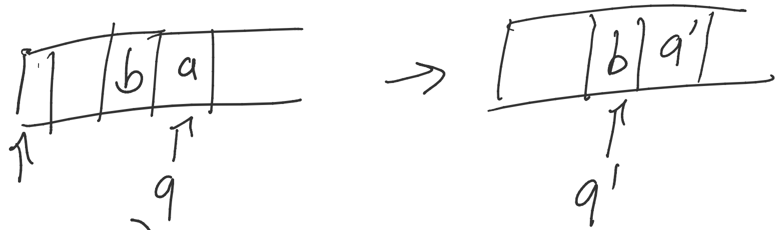
If $\delta(q, a) = (q', a', R)$ (special case: $\alpha q a \underset{M}{\rightsquigarrow} \alpha a' q'$)

$$\alpha q a \beta \underset{M}{\rightsquigarrow} \alpha a' q' \beta$$

If $\delta(q, a) = (q', a', L)$ (Special case: $q a \beta \underset{M}{\rightsquigarrow}$ crash)

If $\delta(q, a) = (q', a', L)$ $\quad (q\, a\, \beta \underset{M}{\rightsquigarrow} \text{crash})$

$$\alpha\, b\, q\, a\, \beta \underset{M}{\rightsquigarrow} \alpha\, q'\, b\, a'\, \beta$$



Def: (Multiple Moves)

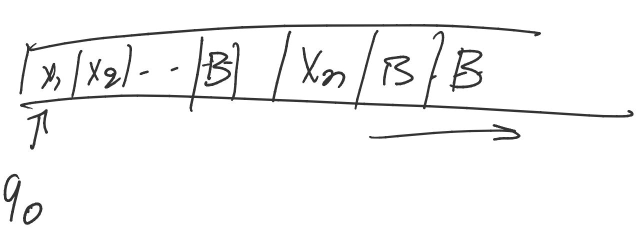
$C \overset{*}{\underset{M}{\rightsquigarrow}} C'$ if $C \underset{M}{\rightsquigarrow} C_1 \underset{M}{\rightsquigarrow} C_2 \cdots \underset{M}{\rightsquigarrow} C_{k-1} \underset{M}{\rightsquigarrow} C'$

where $C_1, C_2, \ldots, C_{k-1}$ are some configurations.

Def: On input $x \in \Sigma^*$

- M accepts x iff $q_0 x \overset{*}{\underset{M}{\rightsquigarrow}} \alpha\, q_{acc}\, \beta$, $\alpha, \beta \in \Gamma^*$

- M rejects x iff $q_0 x \overset{*}{\underset{M}{\rightsquigarrow}} \alpha\, q_{rej}\, \beta$



- M crashes on x iff $q_0 x \overset{*}{\underset{M}{\rightsquigarrow}}$ crash

- M does not halt iff $q_0 x \underset{M}{\rightsquigarrow} C_1 \underset{M}{\rightsquigarrow} C_2 \underset{M}{\rightsquigarrow} C_3 \cdots$ infinite seq.

- M does not halt … infinite seq.

Def: $L(M) = \{ x \in \Sigma^* \mid M \text{ accepts } x \}$

lang. accepted/computed by M.

- L is recursively enumerable (r.e.) iff $L = L(M)$ for some TM M.

- L is recursive (also called decidable) iff $L = L(M)$ for some TM M that halts on all inputs $(\forall x \in \Sigma^*)$

Rmk: Diagram Notation

$q \xrightarrow{a/a',D} q'$ $\qquad \delta(q,a) = (q', a', D)$

$D \in \{L, R, S\}$

no arc $\qquad \delta(q,a) = (q_{rej}, a, S)$

Ex: $\{ 0^n 1^n 2^n \mid n \geq 0 \}$

X X X X Y Y Y Y Z Z Z Z

$\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{1}\cancel{1}\cancel{1}\cancel{1}\cancel{2}\cancel{2}\cancel{2}\cancel{2}$

q_0

Idea: Repeat.

1. Change first 0 to X, go R until find 1.

trick: Marking

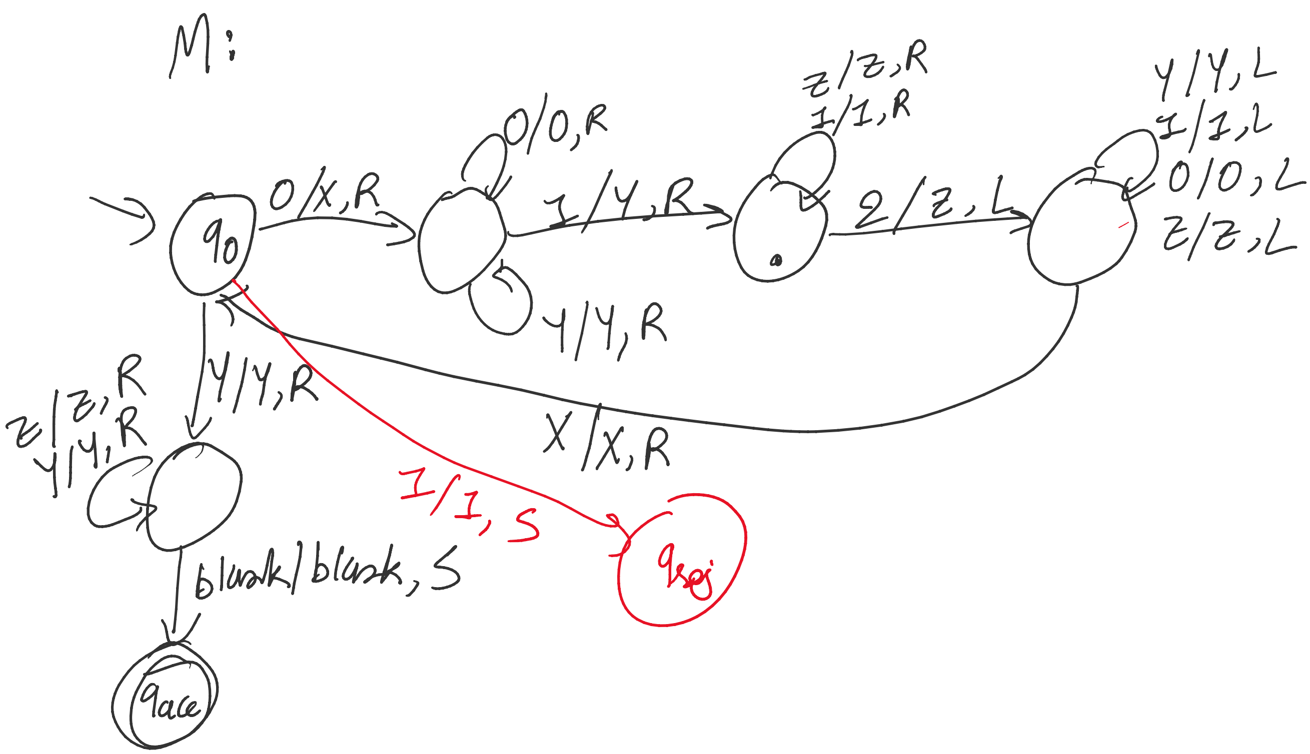
1. Change first 0 to X, go R until find 1.
2. Change 1 to Y, go R until find 2.
3. Change 2 to Z, go L until find rightmost X.

Z

Until no more 0's.

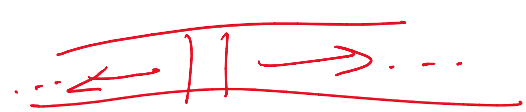
Check no more 1's & 2's

M:



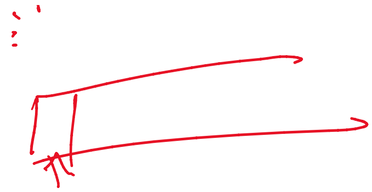
Extensions:

- 2-way infinite tap



- Multiple tapes

tape k



- Non-determinism
- Random access

(can be simulated by the simple "TM.

(may take more time & space)

✱ Church-Turing Thesis :

Any language / function can be solved by some systematic procedure i.e. algorithm iff it can be accepted / computed by a TM.

Rmk: "Thesis" (and not a Theorem) because can't be proved mathematically.

Think of it as "axiom" / "def" of computable".

(So far no counter example!)