Thursday, February 10, 2022 10:50 AM

Last lec : Prove Non-regularity

Fooling set.

- Combining with closure properties

Ex: $L = \{x \in \{0,1,2\}^* \mid \#0(x) \neq \#1(x)\}$

Pf: Suppose L is regular.

Then $\bar{L} = \{x \in \{0,1,2\}^* \mid \#0(x) = \#1(x)\}$

Then $\bar{L} \cap (1+0)^* = \{x \in \{0,1\}^* \mid \#0(x) = \#1(x)\}$

is regular. Contradiction! $\square$

(Skipped) RMK: Applications of Myhill-Nerode Thm.

Test if 2 DFAs are equiv.

2 reg. are equiv.

Pumping Lemma to show non-reg.

☆ Context-Free Language (CFL).

(lang. generated by recursive rules)

Ex: (i) $L = \{0^n 1^n \mid n \geq 0\}$

Rule: $\boxed{S \to 0S1 \mid \varepsilon}$

eg. 000111

$S \rightsquigarrow 0S1 \rightsquigarrow 00S11 \rightsquigarrow 000S111$

$S \to \varepsilon$

$S \rightsquigarrow 0S1 \cdots$

$\Downarrow$

000111.

(ii) $L = \{$ all palindromes in $\{0,1\}^* \}$

$(\{w \mid w = w^R\})$

eg. 01110

$$S \to CSC \mid \varepsilon \mid 0 \mid 1$$
$$C \to 0 \mid 1$$

$$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$$

$$S \rightsquigarrow 0S0 \rightsquigarrow 01S10 \rightsquigarrow 01\,1\,10$$

Definition: A context-free Grammar (CFG) is specified as $G = (\Sigma, \Gamma, P, S)$

where $\Sigma =$ finite alphabet set (terminals) (T)

$\Gamma =$ finite set of variables (non-terminals) (V)

$\Gamma \cap \Sigma = \emptyset$

$P =$ finite set of rules (Production) each of the form

$A \to \alpha$ where $A \in \Gamma$, $\alpha \in (\Gamma \cup \Sigma)^*$

$S \in \Gamma$ is start symbol.

Ex (i) $\Sigma = \{0,1\}$, $\Gamma = \{S\}$

$P = \{ S \to 0S1,\ S \to \varepsilon \}$

Note: - $\alpha_1 \rightsquigarrow \alpha_2$ (α_1 derives α_2 in 1 step)

Def: - $\alpha_1 \underset{G}{\rightsquigarrow} \alpha_2$ (α_1 derives α_2 in 1 step)

if $\alpha_1 = \beta A \gamma$ for some $\beta, \gamma \in (\Gamma \cup \Sigma)^*$

$\alpha_2 = \beta \alpha \gamma$ & $\boxed{A \to \alpha \text{ is in } P}$

- $\alpha_1 \overset{K}{\underset{G}{\rightsquigarrow}} \alpha_2$ (α_1 derives α_2 in k steps)

iff $\begin{cases} \alpha_1 = \alpha_2 & \text{if } K=0 \\ \alpha_1 \underset{G}{\rightsquigarrow} \beta \ \& \ \beta \overset{(K-1)}{\underset{G}{\rightsquigarrow}} \alpha_2 & \text{if } K>0 \end{cases}$

for some $\beta \in (\Gamma \cup \Sigma)^*$

- $\alpha_1 \overset{*}{\underset{G}{\rightsquigarrow}} \alpha_2$ if $\alpha_1 \overset{K}{\underset{G}{\rightsquigarrow}} \alpha_2$ for some $K \geq 0$

Def: $L(G) = \{x \in \Sigma^* \mid S \overset{*}{\underset{G}{\rightsquigarrow}} x\}$ Language generated by G.

Def: L is CFL if $L = L(G)$ for some CFG G.

Ex: a) $\underbrace{(1+\underline{10})^* (1+\underline{01})^*}_{A \quad B} + \underbrace{1^*0}_{C}$,

$S \to AB \mid C$

$A \to 1A \mid \underline{10A} \mid \varepsilon$

$B \to 1B \mid \underline{01B} \mid \varepsilon$

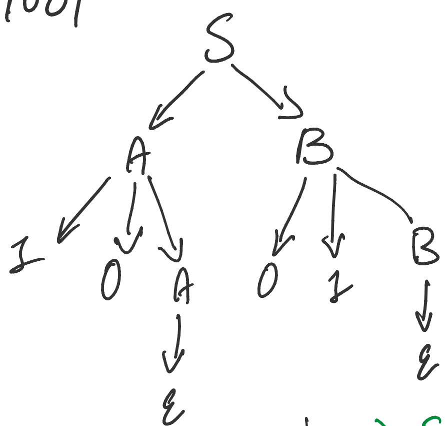
$C \to 1C \mid 0$

[More generally any regular lang. is CFL]

eg. 1001

[More

eg. 1001



"Parse tree"
"derivation tree"

b) $\{0^i 1^j 2^k \mid j > i+k\}$

$\hookrightarrow \underbrace{0^i 1^i}_{A} \; \underbrace{1^{(j-(i+k))>0}}_{B} \; \underbrace{1^k 2^k}_{C}$

$S \rightarrow ABC$

$A \rightarrow 0A1 \mid \varepsilon$

$C \rightarrow 1C2 \mid \varepsilon$

$B \rightarrow 1B \mid 1$

c) $\{0^i 1^i 2^i \mid i \geq 0\}$

impossible!

d) $\{ww \mid w \in \{0,1\}^*\}$

impossible.

e) $\{x \in \{0,1\}^* \mid x \text{ is "not" a palindrom}\}$

eg. $10\,0\,\underbrace{0110}_{\text{anything}}\,101$ $\quad (0+1)^*$

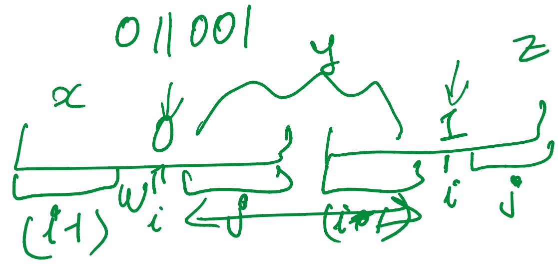
$S \rightarrow 0S1 \mid 1S1 \mid 1A0 \mid 0A1$

$\underline{A} \rightarrow 1A \mid 0A \mid \varepsilon$

f) Complement of $\{ww \mid w \in \{0,1\}^*\}$

011001

y z



g) All strings of balanced parentheses.

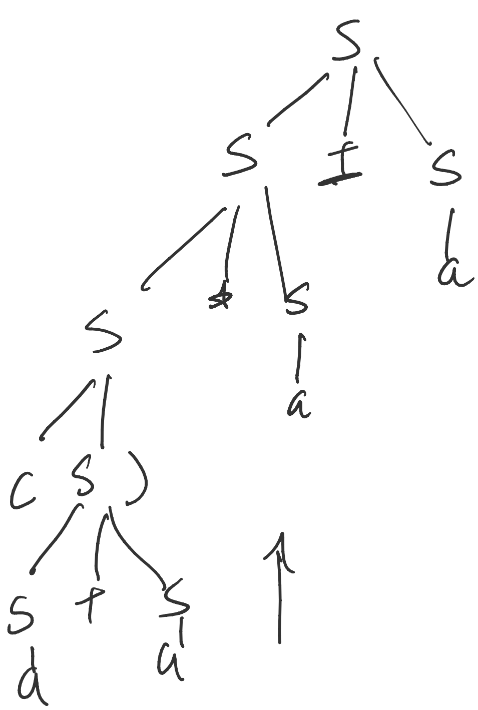
eg. () . (() ())   (()()) (()())

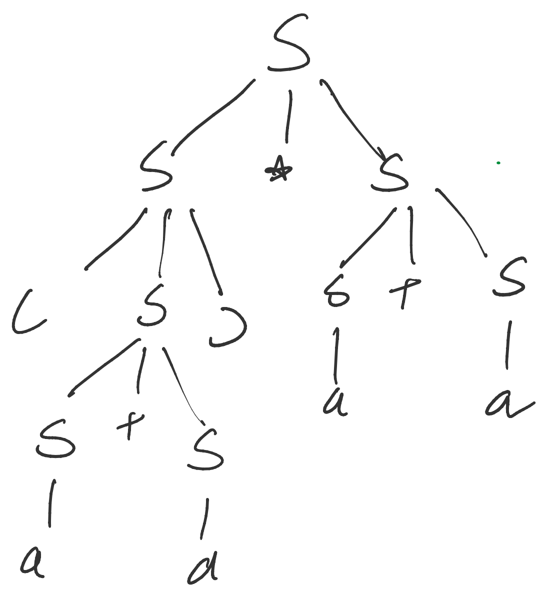
$S \rightarrow (S) \mid SS \mid \varepsilon$

i) All arithmetic ops.

eg. $(a+a)*a+a$

$S \rightarrow S+S \mid S*S \mid (S) \mid a$





Ambiguous

Alternate Sol^n

⇒ Alternate Sol^n

$$S \to S + T \mid T$$

$$T \to T * F \mid F$$

$$F \to (S) \mid a$$

unambiguous.