Prove that each of the following problems is NP-hard.

1. Given an undirected graph $G$, does $G$ contain a simple path that visits all but 374 vertices?
2. Given an undirected graph $G$, does $G$ have a spanning tree with at most 374 leaves?
3. Recall that a 5-coloring of a graph $G$ is a function that assigns each vertex of $G$ a “color” from the set \{0, 1, 2, 3, 4\}, such that for any edge $uv$, vertices $u$ and $v$ are assigned different “colors”. A 5-coloring is careful if the colors assigned to adjacent vertices are not only distinct, but differ by more than 1 (mod 5). Prove that deciding whether a given graph has a careful 5-coloring is NP-hard. (Hint: Reduce from the standard 5COLOR problem.)

![A careful 5-coloring.](image)

Figure 1: A careful 5-coloring.