A subsequence of a sequence (for example, an array, linked list, or string), obtained by removing zero or more elements and keeping the rest in the same sequence order. A subsequence is called a substring if its elements are contiguous in the original sequence. For example:

- SUBSEQUENCE, USEQU, and the empty string $\varepsilon$ are all substrings (and therefore subsequences) of the string SUBSEQUENCE;
- SBSQNC, SQUEE, and EEE are all subsequences of SUBSEQUENCE but not substrings;
- QUEUE, EQUUS, and DIMAGGIO are not subsequences (and therefore not substrings) of SUBSEQUENCE.

Describe recursive backtracking algorithms for the following problems. Don’t worry about running times.


For example, given the array 

$$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 7)$$

your algorithm should return the integer 6, because $(1, 4, 5, 6, 8, 9)$ is a longest increasing subsequence (one of many).


For example, given the array 

$$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 7)$$

your algorithm should return the integer 5, because $(9, 6, 5, 4, 2)$ is a longest decreasing subsequence (one of many).


For example, given the array 

$$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 7)$$

your algorithm should return the integer 17, because $(3, 1, 4, 1, 5, 2, 6, 5, 8, 7, 9, 3, 8, 4, 6, 2, 7)$ is a longest alternating subsequence (one of many).

To think about later:


For example, given the array 

$$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 7)$$

your algorithm should return the integer 6, because $(3, 1, 2, 5, 9)$ is a longest convex subsequence (one of many).
Given an array $A[1..n]$, compute the length of a longest *palindrome* subsequence of $A$. Recall that a sequence $B[1..\ell]$ is a *palindrome* if $B[i] = B[\ell - i + 1]$ for every index $i$.

For example, given the array

$$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 7)$$

your algorithm should return the integer 7, because $(4, 9, 5, 3, 5, 9, 4)$ is a longest palindrome subsequence (one of many).