CS/ECE 374 A (Spring 2022)
Homework 3 (due Feb 10 Thursday at 10am)

Instructions: As in previous homeworks.

Problem 3.1: For each of the following languages in parts (a), (b), and (c), describe an NFA that accepts the language, using as few states as you can. Provide a short explanation of your solution. Below, $\#_0(x)$ and $\#_1(x)$ denote the number of 0's and the number of 1's in x respectively.

(a) (30 pts) all strings $x \in \{0,1\}^*$ such that (x ends with 10101 or 11011) and ($\#_0(x)$ is divisible by 3 or $\#_1(x)$ is divisible by 3).

(b) (30 pts) the language defined by the regular expression $\big(((01)^*0+2)(100)^*1\big)^* \cdot (1^*+0^*2^*)$ over the alphabet $\{0,1,2\}$.

(c) (10 pts) all strings in $\{0,1\}^*$ that contains the pattern 0?1?0, where "?" denotes "don't care" (i.e., a single symbol that is either 0 or 1); in other words, the language defined by the regular expression $(0+1)^* \cdot 0(0+1)1(0+1)0 \cdot (0+1)^*$.

(d) (30 pts) Convert your NFA from part (c) to a DFA by using the subset construction (i.e., power set construction). [*Note*: don't include unreachable states; also, several accepting states can be collapsed into one in this DFA.]

Problem 3.2: Given a language L over the alphabet Σ, define

$$\textsc{move-back}_8(L) = \{xayz : \ xyaz \in L, \ x,y,z \in \Sigma^*, \ a \in \Sigma, \ |y| \le 8\}.$$

Prove that if L is regular, then $\textsc{move-back}_8(L)$ is regular.

(For example, if $010010100110011 \in L$, then $011001010010011 \in \textsc{move-back}_8(L)$.)[1]

[*Hint*: given an NFA (or DFA) for L, construct an NFA for $\textsc{move-back}_8(L)$. Give a formal description of your construction. Provide an explanation of how your NFA works, including the meaning of each state. A formal proof of correctness of your NFA is not required.]

[1]...and also $010101010010011 \in \textsc{move-back}_8(L)$, and $010011010010011 \in \textsc{move-back}_8(L)$, ..., $010010100110011 \in \textsc{move-back}_8(L)$.

For a different example: $\textsc{move-back}_8(0^*1^*) = 0^*1^* + 0^*101^* + 0^*1001^* + 0^*10^31^* + \cdots + 0^*10^81^*$.