

Lecture 1

Administration

A Section:
Beginning - Mar 16 : Patrick
Mar 18 - End: Chandra
Chat Moderator: Vasilis

Review Session
Chandra
Patrick
Fri 9:00
- 10:30a

A vs B: different lecturers
Same Lab/HW/Exams
groups across sections ok!
Labs: Start today / tomorrow.
HW 0 / Quiz 0: available soon ^{now!}

Quiz: due Mon 10a
HW: due Wed 10a
Exams: synchronous,
proctored via Zoom
MT1: Mar 1 6:30p
MT2: Apr 12 6:30p
conflicts
next day
early morn

DRES: Send LOA to Patrick & Chandra well before Midterm

Webpage:

Lecture Schedule / Readings / Recordings

Labs, HW

Office Hours Schedule / Procedures

(Too many) policies

Other Sites:

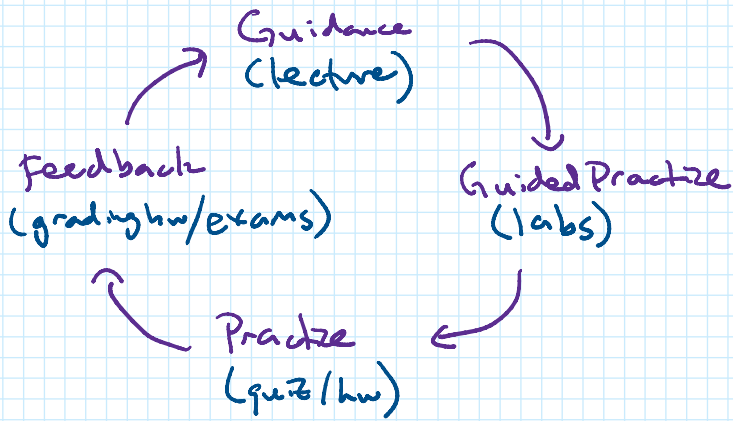
Piazza: Q & A + Announcements

Discord: OH + Socialization

Gradescope: HW/Exam grading

PrairieLearn: Quizzes / Additional Practice

Course Structure / Philosophy



A lot of material very quickly
 refresh prereqs CS 173/225
 → HW0/QUIZ0

What are we talking about?

- Computation, formally (automata, TMs)
- How to compute "efficiently" (algorithm design)
- Limits of computation (undecidability & NP-Hardness)

Key Idea: Reduction $A \leq B$

Informally: problem A, convert it to problem B
 apply known solutions for B to solve A.

Why?

- Computers existed for millennia
 many physical computers share "same" limitations
 capture this into abstract "idealized" mathematical machine
 physical world $\xrightarrow{\text{inspire}}$ model of computation
 model of computation $\xrightarrow{\text{implement}}$ physical world

- efficiency is important & difficult
 e.g. multiplying 2 n-digit #s.

$$\begin{array}{r}
 3141 \\
 \times 2718 \\
 \hline
 25128 \\
 3141 \\
 \hline
 \dots 007
 \end{array}$$

"alternatives"
 e.g. lattice multiplication

$$\begin{array}{r}
 | \\
 \dots 10 +
 \end{array}$$

$$\begin{array}{r}
 25148 \\
 3141 \\
 21987 \\
 6282 \\
 \hline
 8537238 \\
 O(n^2)
 \end{array}$$

equivalent
either asymptotically
or algorithmic

Kolmogorov conjectured $\Omega(n^2)$

Karatsuba 1960 $O(n^{1.58})$

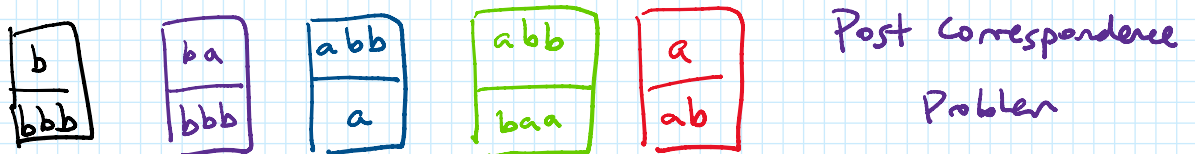
⋮

Schönhage - Strassen 1971 $O(n \log n \log \log n)$

Furer 2008 $O(n \log n 2^{O(\log^* n)})$

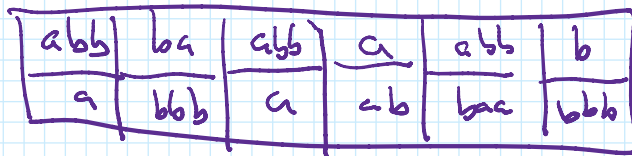
Harvey - van der Hoeven 2019 $O(n \log n)$

- Limits of computation also useful, & surprising.



Post Correspondence Problem

Can I arrange copies of these tiles so the top matches the bottom?



Cannot be solved w/ a computer???

How?

common to many "computers" throughout history to store information as "strings"

Formally: alphabet Σ finite set

$\sigma \in \Sigma$

Formally : alphabet Σ finite set

e.g. $\Sigma = \{0, 1\}$

$$\Sigma = \{a, \dots, z\}$$

$$\Sigma = \{\langle A440 \rangle, \langle C261.63 \rangle, \dots\}$$

string is a finite sequence of symbols from Σ

01001111

'string'

' $\langle C261.63 \rangle \langle F\#369.99 \rangle \langle G392.00 \rangle$ '

Simplistically talk about "Decision" problem
answer is Y/N.

- Post Correspondence Problem

- Given a list, is it sorted?

Language : a set of strings not necessarily finite.

$$L = \{x \mid \text{the answer in problem } P \text{ for } x \text{ is "Yes"}\}$$



P: Given x, is $x \in L$?

deciding Y/N problems = solving "language membership"

Defining strings recursively

a string is either

$$\begin{array}{l} \varepsilon \quad (\text{the empty string}) \\ a \cdot x \quad a \in \Sigma, x \text{ string} \end{array}$$



$$\underline{\text{string}} = s \cdot \text{string} = s \cdot (t \cdot \text{ring}) = \dots = s \cdot (t \cdot (r \cdot (i \cdot (n \cdot (g \cdot \varepsilon))))))$$

operations on strings

e.g. length |string| = 6.

operations on strings

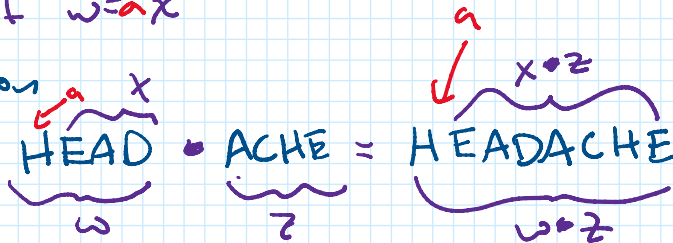
e.g. $\text{length}(\text{string}) = 6.$

$|w| = \# \text{ symbols in } w$

$$|w| = \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = ax \end{cases}$$

e.g. concatenation

$w \circ z$



$$w \circ z = \begin{cases} z & w = \varepsilon \\ a(x \circ z) & w = ax \end{cases}$$

Thm. $|w \circ z| = |w| + |z|$

Pf. Let w, z be arbitrary strings

IH: Assume $|x \circ z| = |x| + |z|$ for all x
where $|x| < |w|$

There are two cases

$$\begin{aligned} \bullet w = \varepsilon &\rightarrow |w \circ z| = |\varepsilon \circ z| && w = \varepsilon \\ \text{(base case)} &= |z| && \text{def of } \circ \\ &= 0 + |z| && \text{add } 0 \\ &= |\varepsilon| + |z| && \text{def of } | \cdot | \\ &= |w| + |z| && w = \varepsilon \end{aligned}$$

$$\begin{aligned} \bullet w = ax &\rightarrow |w \circ z| = |ax \circ z| && w = ax \\ \text{(inductive case)} &= |a(x \circ z)| && \text{def of } \circ \\ &= 1 + |x \circ z| && \text{def of } | \cdot | \\ &= 1 + |x| + |z| && \text{IH} \\ &= |ax| + |z| && \text{def of } | \cdot | \\ &= |w| + |z| && w = ax \quad \square \end{aligned}$$

Ops on Languages

given langs L, M

$$L \circ M = \{x \circ y \mid x \in L, y \in M\}$$

$L \times W = \{(x, y) \mid x \in L, y \in W\}$
 \Rightarrow not a language

$$\begin{aligned} \Sigma^0 &= \{\epsilon\} \\ \Sigma^1 &= \{a \mid a \in \Sigma\} \\ \text{for } n \geq 1, \Sigma^n &= \sum_{i=1}^n \Sigma^{n-i} \end{aligned} \quad \epsilon \neq \{\epsilon\} \neq \{\emptyset\} \neq \emptyset$$

is the set of length n strings

$$\begin{aligned} \Sigma^* &= \bigcup_{n \geq 0} \Sigma^n \quad \text{is the set of all possible strings} \\ &= \bigcup_{n=0}^{\infty} \Sigma^n = \Sigma^0 \cup \Sigma^1 \cup \dots \cup \Sigma^{\infty} \quad \text{countably infinite set} \\ & \quad |\Sigma^*| = |\mathbb{N}| \end{aligned}$$

$$L \subseteq \Sigma^* \quad \text{for all langs } L.$$

every program \Rightarrow a string

$$\# \text{ programs} = |\Sigma^*|$$

$\#$ decision problems

$$= \# \text{ languages} = |\mathcal{P}(\Sigma^*)| \neq |\Sigma^*|$$

exist (uncountably many) problems

that cannot be solved via a computer.