1. Given a graph $G = (V, E)$, a vertex cover of $G$ is a subset $S \subseteq V$ of vertices such that for each edge $e = (u, v)$ in $G$, $u$ or $v$ is in $S$. That is, the vertices in $S$ cover all the edges. Given a tree $T = (V, E)$ and a non-negative weight $w(v)$ for each vertex $v \in V$, give an algorithm that computes the minimum weight vertex cover of $T$; the weight of a cover $S$ is the sum of the weights of the vertices in $S$. In the tree below, $\{B, E, G\}$ is a vertex cover while $\{C, E, F\}$ is not a vertex cover. It is helpful to root the tree.

2. A basic arithmetic expression is composed of characters from the set $\{1, +, \times\}$ and parentheses. Almost every integer can be represented by more than one basic arithmetic expression. For example, all of the following basic arithmetic expression represent the integer 14:

$$
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1
$$

$$
((1 + 1) \times (1 + 1 + 1 + 1)) + ((1 + 1) \times (1 + 1))
$$

$$
(1 + 1) \times (1 + 1 + 1 + 1 + 1 + 1)
$$

$$
(1 + 1) \times (((1 + 1 + 1) \times (1 + 1)) + 1)
$$

Describe and analyze an algorithm to compute, given an integer $n$ as input, the minimum number of 1’s in a basic arithmetic expression whose value is equal to $n$. The number of parentheses doesn’t matter, just the number of 1’s. For example, when $n = 14$, your algorithm should return 8, for the final expression above. The running time of your algorithm should be bounded by a small polynomial function of $n$.

3. To think about later: Suppose you are given a sequence of integers separated by $+$ and $-$ signs; for example:

$$
1 + 3 - 2 - 5 + 1 - 6 + 7
$$

You can change the value of this expression by adding parentheses in different places. For example:

$$
1 + 3 - 2 - 5 + 1 - 6 + 7 = -1
$$

$$
(1 + 3 - (2 - 5)) + (1 - 6) + 7 = 9
$$

$$
(1 + (3 - 2)) - (5 + 1) - (6 + 7) = -17
$$

Describe and analyze an algorithm to compute, given a list of integers separated by $+$ and $-$ signs, the maximum possible value the expression can take by adding parentheses. Parentheses must be used only to group additions and subtractions; in particular, do not use them to create implicit multiplication as in $1 + 3(-2)(-5) + 1 - 6 + 7 = 33$. 