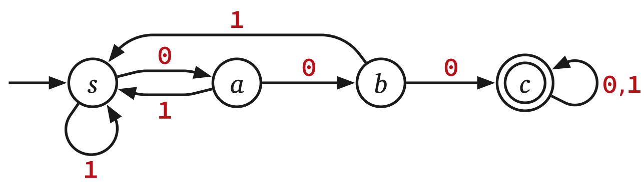
Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{0, 1\}$. Describe briefly what each state in your DFAs *means*.

1. All strings containing the substring 000.

   Solution:

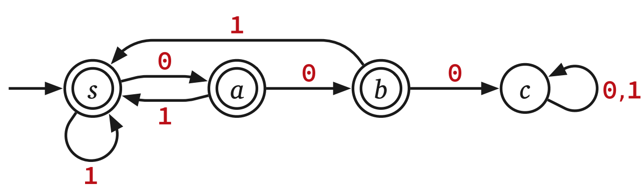
   

   - s: We didn't just read a 0
   - a: We've read one 0 since the last 1 or the start of the string.
   - b: We've read two 0s since the last 1 or the start of the string.
   - c: We've read the substring 000.

   ■

2. All strings *not* containing the substring 000.

   Solution:

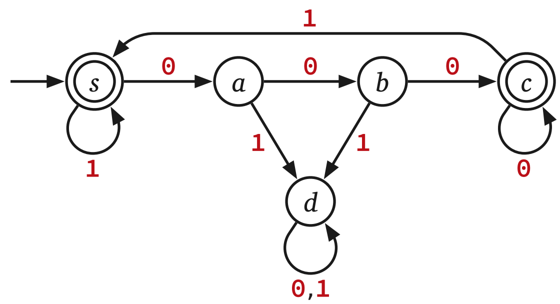
   

   - s: We didn't just read a 0
   - a: We've read one 0 since the last 1 or the start of the string.
   - b: We've read two 0s since the last 1 or the start of the string.
   - c: We've read the substring 000.

   (Yes, these are the same states as in problem 1.) ■

3. All strings in which every run of `0`s has length at least 3.

   Solution:

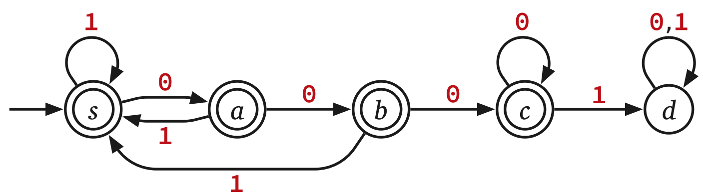


- s: We did not just read a `0`
- a: We've read one `0` since the last `1` or the start of the string.
- b: We've read two `0`s since the last `1` or the start of the string.
- c: We've read at least three `0`s since the last `1` or the start of the string.
- d: We've read the substring `01` or `001`; reject.

■

4. All strings in which no substring `000` appears before a `1`.
   (Equivalently: All strings in which every substring `000` appears after every `1`.)

   Solution: A string is in this language if and only if it does not contain the substring `0001`.

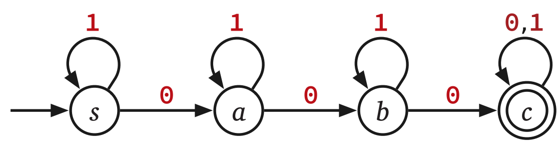


- s: We did not just read a `0`
- a: We've read one `0` since the last `1` or the start of the string.
- b: We've read two `0`s since the last `1` or the start of the string.
- c: We've read at least three `0`s since the last `1` or the start of the string
- d: We've read the substring `0001`; reject.

■

5. All strings containing at least three 0s.

Solution:

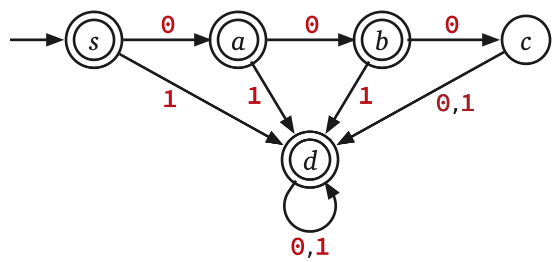


- s: We've read no 0s.
- a: We've read one 0.
- b: We've read two 0s.
- c: We've read at least three 0s; accept.

■

6. Every string except 000. *[Hint: Don't try to be clever.]*

Solution:



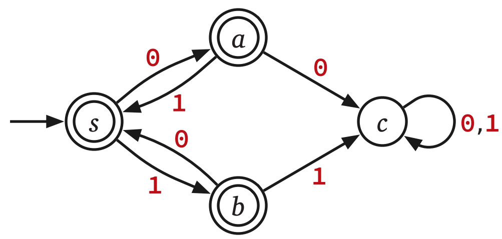
- s: We haven't read anything yet
- a: Input so far is 0.
- b: Input so far is 00.
- c: Input so far is 000.
- d: Input is not 000; accept.

■

Work on these later:

7. All strings w such that *in every prefix of w*, the number of `0`s and `1`s differ by at most 1.

   Solution: This is the same as the set of strings that alternate between `0`s and `1`s.

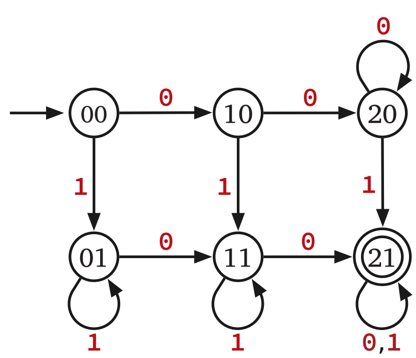
   

   - s: We haven't read anything yet
   - a: Input so far is an alternating string ending in `0`.
   - b: Input so far is an alternating string ending in `1`.
   - c: We've seen the substring `00` or `11`; reject.

   ■

8. All strings containing at least two `0`s and at least one `1`.

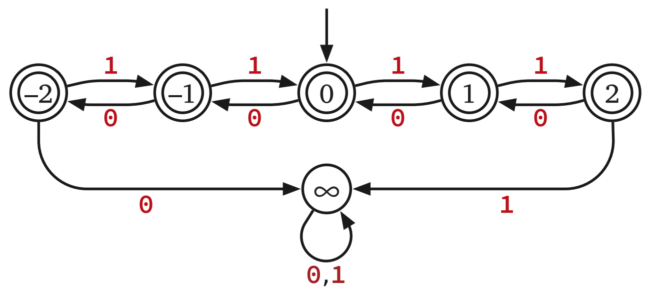
   Solution:

   Each state is labeled with a pair of integers. The first integer indicates the number of `0`s read so far (up to 2), and the second indicates the number of `1`s read so far (up to 1). ■

9. All strings w such that *in every prefix of w*, the number of `0`s and `1`s differ by at most 2.

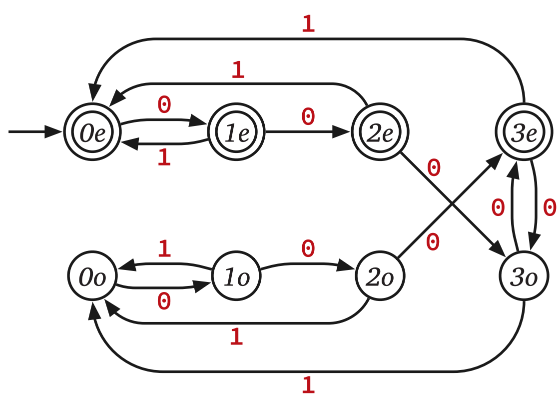
Solution:



The fail state ∞ indicates that we have read some prefix where the number of `0`s and `1`s differ by more than 2. Each of the other states states $-2, -1, 0, 1, 2$ indicates the number of `1`s minus the number of `0`s of the prefix read so far. ■

*10. All strings in which the substring `000` appears an even number of times.
(For example, `0001000` and `0000` are in this language, but `00000` is not.)

Solution:



Each state is labeled with an integer from 0 to 3, indicating how many consecutive `0`s have just been read, and a letter e or o, indicating whether we have read an even or odd number of `000` substrings. ■