CS/ECE 374: Algorithms and Models of Computation, Spring 2021

Midterm 1 – March 1, 2021

- You can do hard things! Grades do matter, but not as much as you may think, but then life is uncertain anyway, so what.
- **Don't cheat.** You know the student code but more importantly you won't be proud later.
- **Please read the entire exam before writing anything.** There are 7 problems and they are of equal value.
- This is a closed-book exam but you are allowed a 1 page (2 sides) cheat sheet that you have to upload along with your exam.
- You should write your answers on clean and clear white pages. After the exam is over you should upload a scan or a photo to Gradescope. If you have technical difficulties you can send the files via email to the course staff. Keep a copy of your written exam in case there are technical difficulties. Try to use different sheets for different problems to make it easier for processing.
- **You have 150 minutes (2.5 hours) for the exam.** This does not include the time for scanning the exam after finishing the writing. You have up to an additional half hour for scanning.
- Proofs are required only if we specifically ask for them.
- You may state and use (without proof or justification) any results proved in class or in the problem sets unless we explicitly ask you for one.
- Please contact the course staff if you run into difficulties with internet, Zoom, etc. You can send email or call/text the staff.

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1 Regular Expression

Give regular expressions for the following two languages.

- $L_1 = \{w \in \{0,1\}^* \mid w$ does *not* have the substring 111$\}$. The string 11001 is in L_1 but not 111101.
- $L_2 = \{w \in \{0,1\}^* \mid$ all 0 blocks of w are of even length$\}$. Recall a 0 block is a maximal non-empty substring of 0's in the string. The string 0000100 is in L_2 but not 001110.

Briefly explain your regular expressions.

2 DFA Construction

Draw or describe a DFA for the language defined below.

$$L = \{w \in \{0,1\}^* \mid w \text{ ends in } 10 \text{ and } |w| \text{ is odd}\}.$$

Your DFA must have at most 6 states. Label the states and explain them.

3 Regular Expressions

Given a language L over alphabet Σ and string $x \in \Sigma^*$ we define the language insert(L, x) as the set of all strings w where w is obtained by taking a string $w' \in L$ and inserting x in w' at some position. More formally insert$(L, x) = \{uxv \mid uv \in L\}$. For example if $L = \{CS374\}$ and $x = not$ then insert$(L, x) = \{notCS374, CnotS374, CSnot374, CS3not74, CS37not4, CS374not\}$. In this problem, assuming that L is regular, you will derive an algorithm that generates a regular expression r' for insert(L, x) from regular expression r for L.

- For each of the base case of $r = \emptyset, \varepsilon$ and $r = a$, $a \in \Sigma$, write down the regular expression for insert$(L(r), x)$
- Assume $r = r_1 + r_2$ and that r_1' and r_2' are regular expressions for insert$(L(r_1), x)$ and insert$(L(r_2), x)$ respectively. Write a regular expression for insert$(L(r), x)$ terms of r_1, r_2, r_1', r_2', x (you do not have to use all of these).
- Assume $r = r_1 r_2$ and that r_1' and r_2' are regular expressions for insert$(L(r_1), x)$ and insert$(L(r_2), x)$ respectively. Write a regular expression for insert$(L(r), x)$ terms of r_1, r_2, r_1', r_2', x (you do not have to use all of these).
- Assume $r = r_1^*$ and that r_1' is a regular expression for insert$(L(r_1), x)$. Write a regular expression for insert$(L(r), x)$ in terms of r_1, r_1', x (you do not have to use all of these).

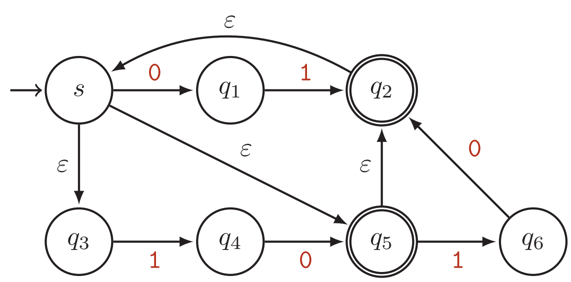
4 NFAs

Let $N_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$ and $N_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$ and $N_3 = (Q_3, \Sigma, \delta_3, s_3, A_3)$ be three NFAs. Let n_1, n_2, n_3 be the number of states in N_1, N_2, N_3 respectively. Describe an NFA $N = (Q, \Sigma, \delta, s, A)$ that accepts the language $(L(N_1) \cap L(N_2)) \cup L(N_3)$. For full credit your NFA N should have $O(n_1 n_2 + n_3)$ states.

- Give a high-level description of how you can construct N using procedures you have learnt in lecture or homework.
- Give a formal tuple notation for N. That is, you need to explain Q, δ, s, A in terms of the parameters of N_1, N_2, N_3.

5 NFAs and Subset Construction

Consider the NFA N shown below.



- Show that 1001 is accepted by N by describing an accepting walk in the NFA.
- What is $\delta^*(s,\varepsilon)$?
- What is the ε-closure of $\{q_3, q_5\}$?
- Consider the subset construction to create a DFA $M = (Q', \Sigma, \delta', s', A')$ from N. What is $\delta'(X, 0)$ where $X = \{q_1, q_2\}$?
- Argue that 100 is not accepted by N by computing $\delta^*(s, 100)$.

6 Non-regularity

Prove that the language $L = \{0^{3^n} \mid n \geq 0\}$ is not regular. You can use any proof technique you want. If you describe a fooling set for the language you need to justify the validity of the fooling set.

7 Regularity

Given a string w a *prefix* is any string x such that there is a string y such that $xy = w$. A proper prefix of w is a string x such that there is y with $|y| \geq 1$ such that $xy = w$. We will call a string x a proper-proper prefix of w if there is a string y such that $|y| \geq 2$ and $xy = w$. If $w = abcde$ then abc, ab, a, and ε are proper-proper prefixes of w. For a language L we define the language

$$\text{PPPREFIX}(L) = \{x \mid x \text{ is proper-proper prefix of some string } w \in L\}$$

Alternatively,

$$\text{PPPREFIX}(L) = \{x \mid \exists y, |y| \geq 2, xy \in L\}.$$

Suppose L is regular. Prove that PPPREFIX(L) is regular. In particular, given a DFA $M = (Q, \Sigma, \delta, s, A)$ for L describe an NFA N that accepts the language PPPREFIX($L(M)$).