Reg Langs & DFA

Tuesday, January 28, 2020
11:49 AM

be regular expressions

\[ R = \varepsilon \]
\[ \{ a \} \]
\[ R_1 R_2 \]
\[ R_1 + R_2 = \overset{\text{all}}{\rightarrow} (R_1 R_2) \]
\[ R^* \overset{\text{any}}{\rightarrow} \{ R \}^* \]
\[ R_1 \cup R_2 \]

ex.
\[ R_0^i (0^i 1^i 0^i) \]

"all strings containing two 1's"?

\[ \Pi \subseteq R \]
\[ 0's \text{ as } 0, \text{ any else plus} \]

EX. "all string with even # of 1's"

\[ (R_{21})^+ \]
\[ 0^* + (R_{21})^+ \]

- REG. LANG. closed under \( \cup \)
- REG. LANG. closed under \( \cap \)
- REG. LANG. closed under complement

\[ R = \{ \varepsilon | w \in \Sigma^*, w \in R \} \]

strings and "0 1 1" \((01)^* 011 (01)^*\)

EX. \( \{ 0^n 1^n | n \in \mathbb{N} \} \) is not regular

- \( 3's \) and \( 3's \)
- \((0^* 1^* 0^* 1^*)^\mathbb{N} \)

01 00 11

DFA and State Machines

Deterministic Finite State Automata
easier to regular expressing

State

day \rightarrow \text{night}

start \rightarrow \text{stop}

start \rightarrow \text{sleep}

Students in 374

feedback \rightarrow \text{ask key}

\text{paying attention}

State machine for recognizing \( E = \{0, 1\} \)

\( M = \)

\text{start} \rightarrow \text{sym 1

\text{walk (M)} \rightarrow \Theta \rightarrow 0

\text{accepting states}

\forall x \in L(M)

\text{if walk}(M, x) \in \text{Accept States

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for i in |x|:
    n_ones += 1 if x[i] = 1
return 1 if n_ones \times 2 = 0
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Ex.

 setState(q3)

 $\begin{array}{c}
 \text{q} \text{, q}_3 \text{, q}_2 \text{, q}_2 \\
 \end{array}$

"Can't have consecutive zeros" $\times$

$$\frac{(01) + (10)}{1001}$$

1001

"All binary strings that when treated as integers (big endian), that are multiples of S"

$$\frac{1011}{8} = 2$$

1610 = 10 = 0 \text{mod} 5

161 = 5 = 0 \text{mod} 5

Reg exp?
\[
\text{Observation:}
\begin{align*}
W &= a \mod S \\
W_0 &= 2^0 \mod S \\
W_1 &= 2^1 \mod S
\end{align*}
\]

\[
\begin{align*}
101 &= s \\
1011 &= 11
\end{align*}
\]

\[
M = (Q, \Sigma, \delta, q_0, F, A)
\]

- \(Q\) is a finite set \(|Q| \in \mathbb{N}\)
- \(\Sigma\) is a finite set \(|\Sigma| \in \mathbb{N}\)
- \(\delta\) is the transition function
- \(q_0\) is the start state
- \(F\) is the set of final states
- \(A\) is the set of accepting states
0 \cup \{x \leq 1 \Rightarrow \} \\
S \in Q \\
A \subseteq Q \\
\text{walk}(M, w) = s \ldots s(s(s(w_0), w)[3], w[1]) \ldots \\
\text{if } |w| = 0, \text{ then } s \\
\text{else } v = ax \text{ then } \text{walk}'(M, x, s(q, a)) \\
\text{walk}(M, w) = \text{walk}'(M, w, s) \\
\text{Note: } \text{walk}(M, w) \text{ is } \delta^A(w, s) \text{ in Slides} \\
L = \{ w \mid w \in \Sigma^*, w \text{ ends in } "\ldots01" \} \\
\text{Diagram of DFA}

\[ \delta^A : Q \times E \rightarrow Q \]

\[ \delta^A(q, w) = \]
\[ \delta^A(q, \epsilon) = q \]
\[ \delta^A(q, a \cdot x) = \delta^A(\delta^A(q, a), x) \]

\[ L(M) = \{ w \mid w \in \Sigma^* \text{ and } \delta^A((s, w) \in A \} \]

A string \( x \), symbols \( a \), and DFAs \( M = (Q, \Sigma, \delta, q_0, F) \)

**Thm:** \[ \delta^A(q, X, a) = \delta(\delta^A(q, X), a) \]

**Proof:** By induction on \( X \)

Base case: \( X = \epsilon \) \( |X| = 0 \)

Goal: \[ \delta^A(q, \epsilon) = \delta(\delta^A(q, \epsilon), a) \]

\[ = \delta(q, a) \]

\[ \because \text{by base case of defn of } \delta^A \]

Inductive case: \( X = a \) \( |X| = n+1 \) \( N = n \)

IH: \( \forall q. \delta^A(q, y, a) = \delta(\delta^A(q, y), a) \)

Goal: \[ \delta^A(q, a \cdot x) = \delta(\delta^A(q, a \cdot x), a) \]

\[ = \delta(\delta(q, a), xa) \]

Let \( q' = \delta(q, a) \)
By IH  \( S^* (S(a, b), y a) = S(S^*(S(a, b), y a)) \)

\( S(S^*(x, b y), a) \)

by defn \( S^* \)

**Thm:** Let \( N = \langle Q, \Sigma, \delta, q_0, F \rangle \)

\[ L(N) = \{ w \mid \#_1(w) \text{ is even} \} \]

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**Product Construction**

Regular

- \( Q \cap R_1 \)
- \( Q \cup R_1 \)
- \( A \cap R_2 \)
- \( A \cap \overline{R} \)

\( 0^* 1^* \)

\( 0^* 1^* \)

\( 0^* 1^* \)

\( = \text{DFA} \)

\( \overline{\text{DFA}} \)

\( Q \cap \overline{R} \)

\( Q \cup \overline{R} \)

\( Q \cap \overline{R} \)

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Even
\[ M = (\varepsilon, Q, S, s_0, A) \]

\[ \overline{M} = (\varepsilon, Q, S, s_0, \overline{A} = Q - A) \]

\[ L(M) = L(\overline{M}) \quad \text{complement} \checkmark \]

**Product Construction**

\[ \exists w \mid \#_0(w) = \text{even} \quad \text{and} \quad \#_1(w) = \text{even} \]

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\[ M_1 = (Q, \varepsilon, S_1, s_1, A_1) \]
\[ M_2 = (Q_2, \Xi, \delta_2, S_2, A_2) \]
\[ M_{1 \times 2} = (Q_1 \times Q_2, \Xi, \delta_{1 \times 2}, (\delta_1, \delta_2), A_{1 \times 2}) \]
\[ (\delta_1, \delta_2) \in Q_1 \times Q_2 \]
\[ A_{1 \times 2} = A_1 \times A_2 = \{ (q_1, q_2) \in Q_1 \times Q_2 \mid q_1 \in A_1, q_2 \in A_2 \} \]
\[ \delta_{1 \times 2} : (Q_1 \times Q_2) \times \Xi \rightarrow (Q_1 \times Q_2) \]
\[ \delta \left( (q_1, q_2), a \right) = \left( \delta_1 (q_1, a), \delta_2 (q_2, a) \right) \]