Undecidability

Undecidable means a decision problem
that can be solved in any amount of time.
Not exponentially, nothing goes.

Today: how to define an undecidable problem
and prove it.

Simulating Turing machines "universal language"
- TM
- λ calculus
- python
- pseudocode

Notation:
\(< M >\) string encodes the
machine M.
M a machine.

For DFAs: \(M = (\Sigma, \delta, q_0)\)
\(< M > = \langle \ldots \rangle \) encoding of M

Pseudocode is universal:
Let \(\text{Simulate}(< M >)\) be
Simulate the machine M
on input "hello"

Machines might infinite loop
Define $\mathcal{M}(w) \downarrow 0$ as convergence. 

- $\mathcal{M}(w) \downarrow 0$: $M$ halts and returns $0$ 
- $\mathcal{M}(w) \downarrow 1$: $M$ accepts $w$ 
- $\mathcal{M}(w) \uparrow$: $M$ does not halt on $w$ 
- $\mathcal{M}(w) \downarrow$: $M$ halts on $w$

Accepts $Hi = \{<M> : M$ accepts $"hi" ? \}

Accepts $All = \{<M> : M$ slows $x$, $M$ accepts $x \}$

Self-Halt $= \{<M> : M$ halts on $\Sigma^{<M>3} \}$

Thus, Self-Halt is undecidable.

Suppose $SH$ is a solution to Self-Halt.

Let $P(w) = \{
\begin{array}{ll}
\text{run } SH(w) & \\
\text{if YES: } & \text{Directly return YES} \\
\text{else:} & \text{return YES}
\end{array} \}$

Consider $P(<p>)$.

Suppose $P(<p>)$ holds. $SH(<p>)$ must return NO.

Consider $P(<p>)$. $SH(<p>)$ must return NO. 

https://onedrive.live.com/redir?resid=D2ECBD89F9D7F77E%21110&page=Edit&wd=target%28ECE374 S20%2F Lecture scribbles.on
So undecidability still being a deciding factor.

... suppose \( p(\langle p, w \rangle) \) diverges

\[
\text{Reductions:}
HALT = \{ \langle M, w \rangle \mid \exists \langle M', w \rangle \mid M \text{ halts on } w \}
\]

To show halting undecidable.

Self-Halt \leq HALT

\[
\langle M \rangle \xrightarrow{\text{Let } x = \langle M', w \rangle} \langle M' \rangle \xrightarrow{\text{HALT}} \text{Yes/No}
\]

\[
\text{HALT}_{\text{Same}} = \{ \langle M_1, M_2 \rangle \mid \exists x \mid M_1 \text{ halts } \iff M_2 \text{ halts on } x \}
\]

Is this undecidable?

HALT \leq HALT_{\text{Same}}
Given \(<M_1, X_1, \ldots, X_n>\):

- \(M_1(X_1) = 1\)
- \(M_2(Y) = \frac{M_2}{\text{hats}}\)
- \(M_3(\ldots)\)

Rice's Theorem: any non-trivial property is undecidable.