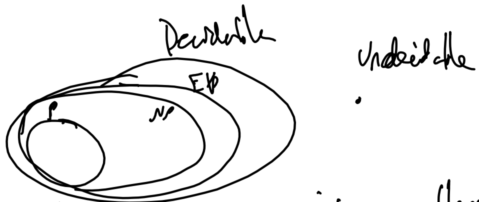
Undecidability

Thursday, April 30, 2020 2:39 PM



Undecidable: means a decision problem that can't be solved in any amount of time.
Not exponential, not even anything greater.

Today: how to define an undecidable problem and prove it.

Simulating Turing machines "universal language"

- TM
- λ-calculus
- python
- pseudocode ☺

Notation:
$\langle M \rangle$ string, encodes the machine M.

M a machine.

For DFAs: $M = (\Sigma, \delta, Q \ldots)$
$\langle M \rangle$ = "_ _ _ _ _" encoding of M

Pseudocode is universal:
Let Simulate($\langle M \rangle$) be
Simulate the machine M
on input "hello"

Machines might infinite loop, like

input string → $M(w) \downarrow 0$ ← converges/halts ← output; machine → M

: $M(w)$ halts and returns 0 — M <u>rejects</u> w

$M(w) \downarrow 1$ — M <u>accepts</u> w

$M(w) \uparrow$ — M does not halt on w; M <u>diverges</u> on w

AcceptsHi = { <M> : M accepts "hi" }
  M("hi") halts and outputs 1

AcceptAll = { <M> : ∀ strings x, M accepts x }

SelfHalt = { <M> : M halts on <M> }

Thm: SelfHalts is undecidable

Suppose SH is a solution to SelfHalts

<M> → [SH] → Yes iff <M> ∈ SelfHalts; → No

Let P(w) be
  run SH(w)
  if YES:
    diverge
  else:
    return YES

Consider P(<P>).

Suppose P(<P>) halts.
SH(<P>) must return NO
By assumption, <P> is self halting.

By assumption ...
So contradicts SH being a decider for SelfHalts,

... Suppose $P(\langle P \rangle)$ diverges

...

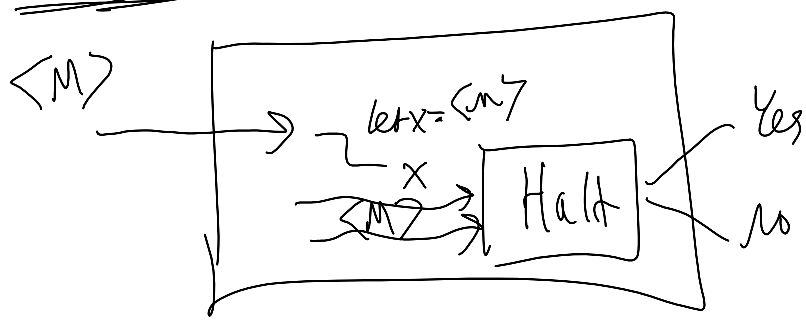
Redu ctions.

$$HALT = \cancel{\{(\langle M \rangle, w)}$$

$$\{\langle M, w \rangle \mid M \text{ halts on string } w\}$$

To show halt undecidable.

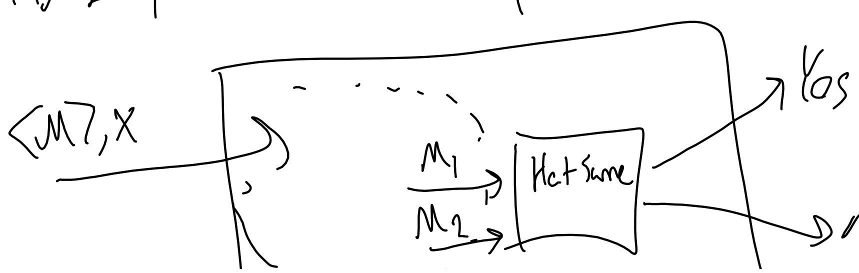
$$\underline{SelfHalt} \leq \underline{Halt}$$



$$HALTSame = \{\langle M_1 \rangle, \langle M_2 \rangle : \forall x \; M_1 \text{ halts on } x \iff M_2 \text{ halts on } x\}$$

Is this undecidable?

$$HALT \leq Haltsame$$



Given $\langle M \rangle, x,$

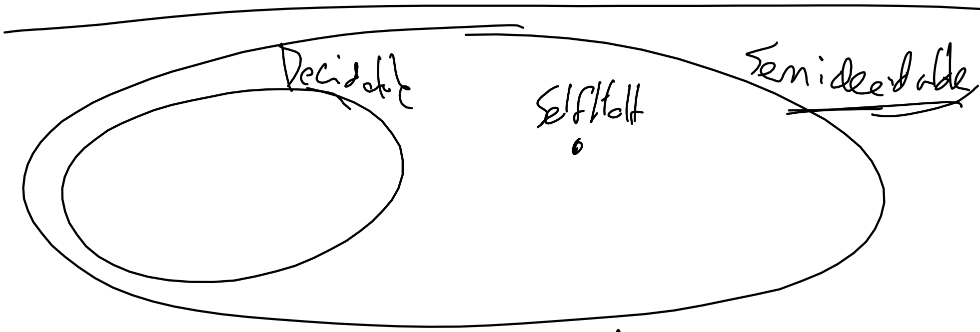
let $M_1(\_)$ run $M(x)$?

let $M_2(y) =$ halts

M_2 ~~never~~ always halts on any input. ✓

Rice's Theorem: any nontrivial property is undecidable

not allowed to use



NP: polytime decidable on yes