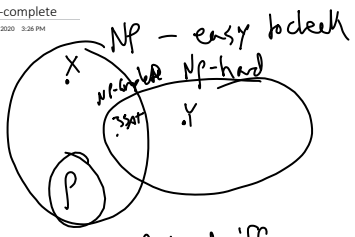
More NP-complete
Tuesday, April 28, 2020 3:26 PM



Y is NP-hard iff
$\forall X \in NP,\ X \leq_p Y$

– Proving a problem is NP-hard
reducing a known NP-hard problem to it

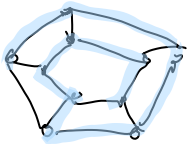
e.g. $3SAT \leq_p Y$

– $3SAT \leq_p$ ind Set. (last time)

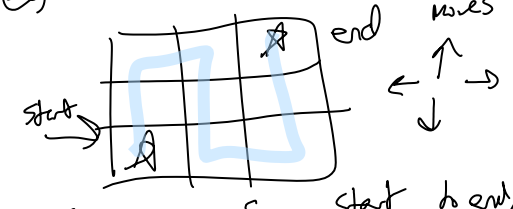
– Claim: 3SAT is NP-hard.
$\Rightarrow \forall X \in NP,\ X \leq_p 3SAT$.
If we show $3SAT \leq_p Y$,
$\forall X \in NP,\ X \leq_p 3SAT \leq_p Y$

Today: $3SAT \leq_p$
– hamiltonian paths
– 3 coloring
– Self reducibility.

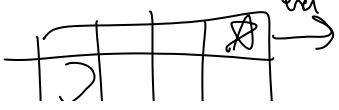
Hamiltonian paths:
– Simple cycle in a graph that visits every node. \ no repeated nodes/edges



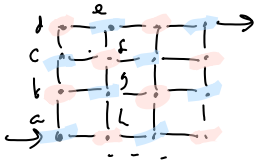
e.g.:



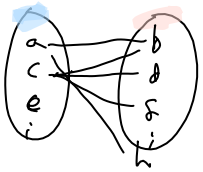
Can you go from start to end, only moving to adjacent sq's and visiting each sq. once



If not solvable, how can you prove it?



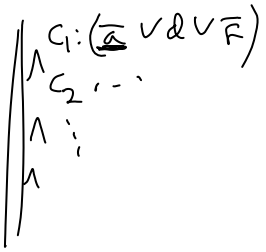
Bipartite:
∃ a 2-partitioning such that all edges are between nodes in diff partitions.



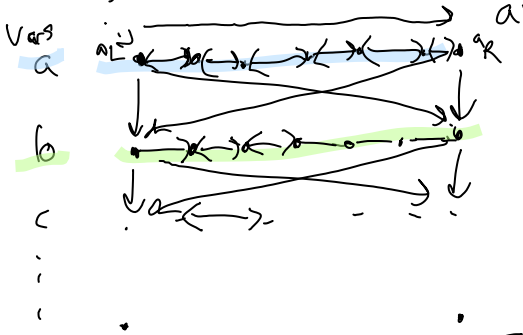
Hamiltonian paths is NP-hard

HAMILTON(G) → {0,1} does it hamiltonian have a path or not

3SAT ≤p HAMILTON

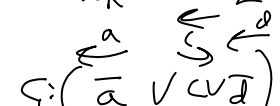


- First, variables (T or F)
- Clause gadgets

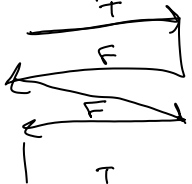


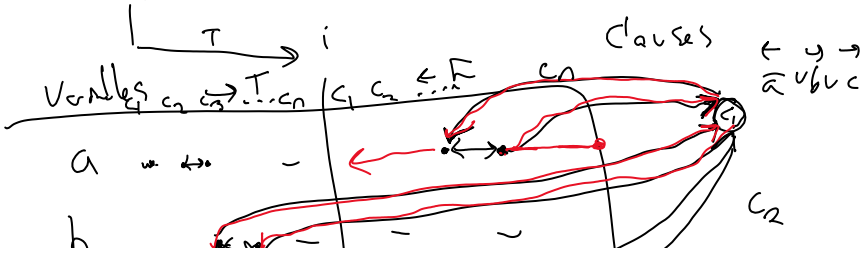
a: True → left to right
a: False ← right to left

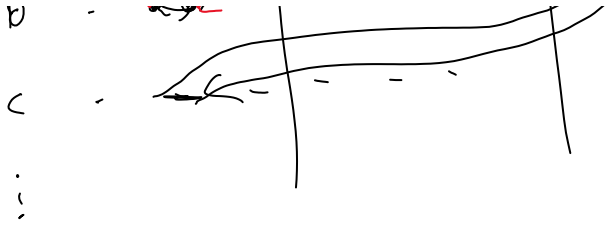
either visit a_L before a_R →
or a_R before a_L ←



Any Hamiltonian path must look like





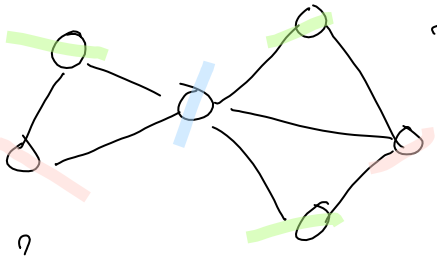


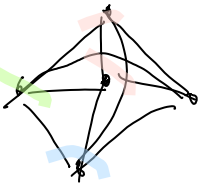
3SAT $\leq_p$ Dir-HAMILTON $\leq_p$ Undirected-HAMILTON

3SAT $\leq_p$ 3-Coloring

2 coloring? X

3-colored? ✓

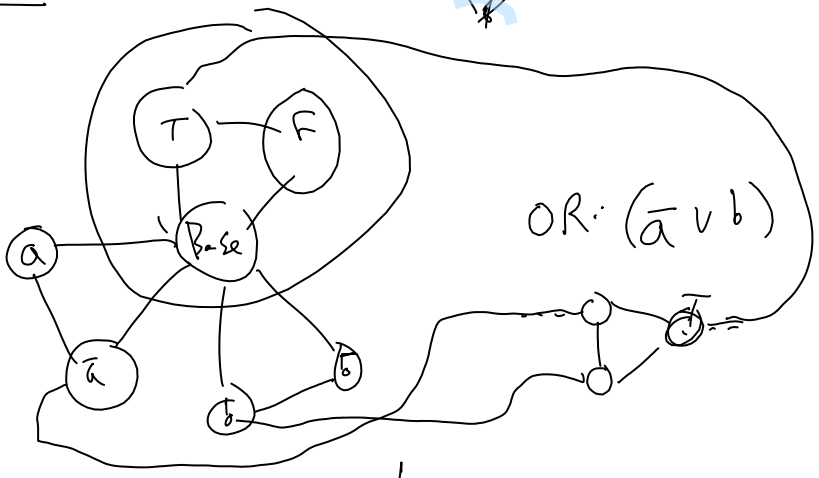




4-coloring?

Planar graphs are 4 colorable.

- Variable values
- Clauses

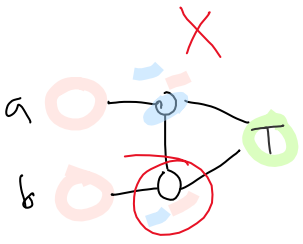


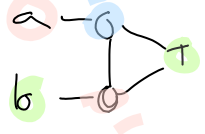
C_1 ⋮

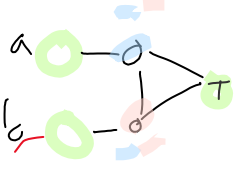
Gadget for each clause.

OR gadget

a
b
T

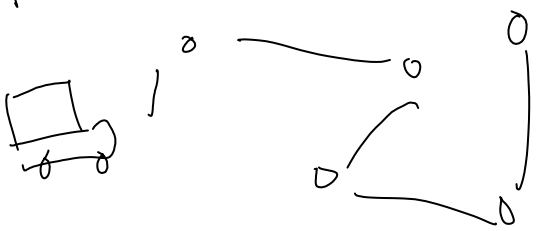






✓

What to do once you've proven your problem is NP-Complete!

TSP 

⇒ You probably can't find a poly time algo..

⇒ give up?

⇒ worst case can't polytime.

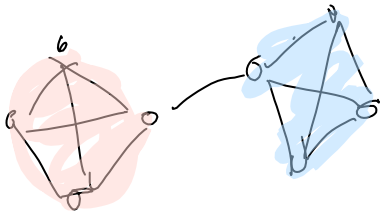
- Might be the case that most, or randomly selected, or other class of instances
- heuristics randomization.
- approximate....
- - - exponential is ok for small instances.

Self-reducibility:

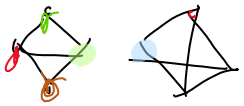
- Often "search" problem reduces to the "decision" problem.

Clique-Cover ⇒ dec CIC

Yes, there's a k clique cover.

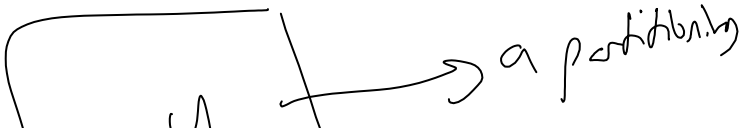


Every graph of n nodes has a n-clic.



Find CIC ≤p dec CIC?

dec CIC (G, k): does G have a k clique cover?




del