NP - easy to check

Y is NP-hard iff
\[ \forall X \in \text{NP}, \quad X \leq_p Y \]

- Problem written is NP-hard
- reducing a known problem to it
  
  e.g., \[ 3\text{SAT} \leq_p Y \]

- \(3\text{SAT} \leq_k \text{3rd Set}\)

- Claim: \(3\text{SAT} \leq \text{NP-hard}\)

- \(\Rightarrow \forall X \in \text{NP}, \quad X \leq \text{3SAT}\)

- We show \(3\text{SAT} \leq_p Y\)

- \(\forall X \in \text{NP}, \quad X \leq_p 3\text{SAT} \leq_p Y\)

Today: \(3\text{SAT} \leq\)
- Hamiltonian paths
- 3 coloring
- Self reducibility.

Hamiltonian paths:
- Simple cycle in a graph that visits every node, no repeated nodes/edges

Example:

Can you go from start hand, only moving to adjacent sq. and visiting each sq. once?
Hamilton paths & cycles

\[ \text{HAMILTON}(G) \rightarrow 0,1 \]

\[ 3SAT \leq_p \text{HAMILTON} \]

- First, variables (T or F)
- Clauses generated

Any Hamilton path must look like

Clauses

Prove if there is one.
$3\text{SAT} \leq_p \text{Di}_{\text{r-}}\text{HAMILTON} \leq_p \text{Undirected-HAMILTON}$

$3\text{SAT} \leq_p 3\text{-Color}$

2-clique? $\times$
3-clique? $\checkmark$

4-clique? no 4-clique.

Planar graphs are 4-colorable.

Variable values
Clause $C_1$:
Gadget for each clause
a or gadget

OR: $(\overline{a} \lor b)$

OR gadget

What to do once you've proven your problem is NP-complete:
You probably want a poly-time solution...

⇒ Give up?

⇒ Worst case can't be poly-time.

- Might be the case that most
  or randomly selected
  or other class of instances
  - heuristics
  - randomized
  - approximate...
  - exponential is ok for small instances.

Self-reducibility:
- Often "search" problem reduces to the "decision" problem.

$\text{CLIQUE-COVER} \Rightarrow \text{dec CLI}

\text{Yes, there's a k-clique cover.}$

Every graph of N nodes has an n-clique.

$\text{Find CLI \leq dec CLI?}$

$a \text{ a polynomial}$
del