HW 8

Due on **Monday, March 30, 2020 at 11pm**

CS/ECE 374 B: Algorithms & Models of Computation, Spring 2020 Version: **1.33**

Submission instructions as in previous homeworks.

22 (100 PTS.) COVID-19

COVID-19 is the most major pandemic of our lifetime! As of 03/23/2020 3 pm CST, more than 370,000 individuals have been affected and the numbers are increasing exponentially fast everyday. Please stay at home and avoid social contact.

Graphs are an extremely powerful tool in analyzing and tracking the spread of viruses. Suppose there are n people in a given community, $P_1, P_2, \cdots, P_n$. You are given the time at which any pair of individuals came into contact during some observation period. Hence, the data is a sequences of ordered triples (P_i, P_j, t_k) which means that person P_i was less than 6 feet away from person P_j at time t_k. Furthermore, if a person P_i carrying the corona virus comes in contact with another person P_j not carrying the corona virus at time t_k, then the virus is transfered to person P_j at time t_k.

The infection can spread from one person to another across a sequence of contacts, provided that no step in this sequence involves a move backward in time. Hence, if P_i contracts the virus at time t_k, and the data you are given contains triples (P_i, P_j, t_k) and (P_j, P_q, t_r), where $t_k \leq t_r$, then P_q will contract the virus via P_j. (Note that it is okay for t_k to be equal to t_r; this would mean that P_j was less than 6 feet away from both P_i and P_q at the same time, and so a virus could move from P_i to P_j to Pq.)

For example, suppose $n = 4$ and person P_1 contracts the virus at time $t = 2$.

If the data contains the triples:

$$(P_1, P_2, 4), (P_2, P_4, 8), (P_3, P_4, 8), (P_1, P_4, 12)$$

Then, P_3 would contract the virus at time 8. However, if the data contains the triples:

$$(P_2, P_3, 4), (P_1, P_4, 8), (P_1, P_2, 12)$$

Then, P_3 would not contract the virus during the observation period.

For simplicity, you can assume that the triples are given to you in a sorted order of time. You can also assume that each pair of individuals come into contact at most once during the observation period and sick individuals remain sick for the entire observation period.

Design an algorithm that given a sequence of m triples and the observation that person P_x contracted the virus at time t_x, finds all the people who would contract the virus during the observations interval and the time at which they contracted the virus. (**Hint:** Build a graph. What are the vertices? What are the edges? What problem is this?)

23 (100 PTS.) **Racetrack**

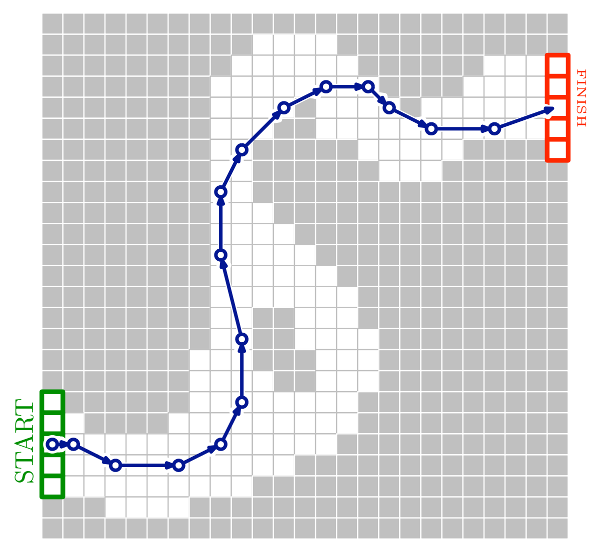
Consider the following two-player paper-and-pencil racing game. The game is played with a track drawn on a sheet of graph paper. The players alternately choose a sequence of grid points that represent the motion of a car around the track, subject to certain constraints explained below.

Each car has a *position* and a *velocity*, both with integer x- and y-coordinates. A subset of grid squares is marked as the *starting area*, and another subset is marked as the *finishing area*. The initial position of each car is chosen by the player somewhere in the starting area; the initial velocity of each car is always $(0, 0)$. At each step, the player optionally increments or decrements either or both coordinates of the car's velocity; in other words, each component of the velocity can change by at most 1 in a single step. The car's new position is then determined by adding the new velocity to the car's previous position. The new position must be inside the track; otherwise, the car crashes and that player loses the race. However, it is not necessary for the line between the old position and the new position to lie entirely within the track. The race ends when the first car reaches a position inside the finishing area.

Suppose the racetrack is represented by an $n \times n$ array of bits, where each 0 bit represents a grid point inside the track, each 1 bit represents a grid point outside the track, the "starting area" is the first column, and the "finishing area" is the last column.

Describe and analyze an algorithm to find the minimum number of steps required to move a car from the starting line to the finish line of a given racetrack. (**Hint:** Build a graph. What are the vertices? What are the edges? What problem is this?)

velocity	position
$(0,0)$	$(1,5)$
$(1,0)$	$(2,5)$
$(2,-1)$	$(4,4)$
$(3,0)$	$(7,4)$
$(2,1)$	$(9,5)$
$(1,2)$	$(10,7)$
$(0,3)$	$(10,10)$
$(-1,4)$	$(9,14)$
$(0,3)$	$(9,17)$
$(1,2)$	$(10,19)$
$(2,2)$	$(12,21)$
$(2,1)$	$(14,22)$
$(2,0)$	$(16,22)$
$(1,-1)$	$(17,21)$
$(2,-1)$	$(19,20)$
$(3,0)$	$(22,20)$
$(3,1)$	$(25,21)$



A 16-step Racetrack run, on a 25×25 track. This is *not* the shortest run on this track.

24 (100 pts.) 374-Walk

Consider a directed graph G, where each edge is labeled with a symbol: either **3**, **7** or **4**.

A walk in G is called a ***374 walk*** if its sequence of edge symbols is **3, 7, 4, 3, 7, 4, 3, 7,** $\cdots$. Formally, a walk $v_0 \to v_1 \to \cdots \to v_k$ is a ***374 walk*** if, for every integer i, the edge $v_i \to v_{i+1}$ is labeled with symbol **3** if $i \bmod 3 = 0$, **7** if $i \bmod 3 = 1$, and **4** if $i \bmod 3 = 2$.

Describe an efficient algorithm to find all vertices in a given labeled directed graph G that can be reached from a given vertex v through a ***374 walk***.