HW 4

Due on **Monday, February 17, 2020 at 11pm**

CS/ECE 374 B: Algorithms & Models of Computation, Spring 2020 Version: **1.01**

Submission instructions as in previous homeworks.

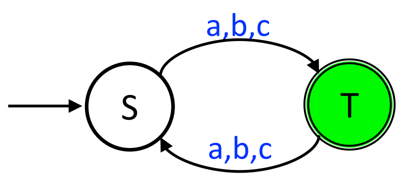
10 (100 PTS.) **Prove Regular**

Let Σ be finite alphabet. A ***code*** is a mapping $f : \Sigma \rightarrow \{0, 1\}^+$. For example, if $\Sigma = \{a, b, c\}$, a code f might be $f(a) = 101$, $f(b) = 01100$, and $f(c) = 10$. (To simplify things, we assume for any $a \neq b$, we have $f(a) \neq f(b)$.)

For a string $w_1 w_2 \cdots w_m \in \Sigma^*$, we define $f(w) = f(w_1)f(w_2)\cdots f(w_m)$. In the above code,

$$f(abcba) = 101 \bullet 01100 \bullet 10 \bullet 01100 \bullet 101. = 101011001001100101.$$

10.A. (10 PTS.) Let L be the language of the following DFA M. What language does L represent ?



10.B. (20 PTS.) Working on the DFA M from **10.A.** construct an NFA for the language $f(L)$. Here, $f(L) = \{f(w) \mid w \in L\}$ is the *code language* where f is code from the above example.

10.C. (70 PTS.) Let $L \subseteq \Sigma^*$ be a arbitrary regular language. Prove that the coded language $f(L) = \{f(w) \mid w \in L\}$ is regular.

Specifically, given a DFA $M = (Q, \Sigma, \delta, s, A)$ for L, describe how to build an NFA N for $f(L)$. Then, prove the correctness of your construction, i.e., the language of the constructed NFA is indeed the desired language $f(L)$. Your construction and proof should be for any arbitrary code f and not just the code in the example above. (30 points for a correct construction, and 40 points for a correct proof of correctness.)

11 (100 PTS.) **Prove Not Regular**

Prove that the following languages in **11.A.** to **11.C.** are not regular by providing a fooling set. You need to prove that it is an infinite fooling and valid fooling set.

11.A. (25 PTS.) $L = \{0^i 1^j 2^k \mid i + j = k + 1\}$.

11.B. (25 PTS.) $L = \{0^{n^3} \mid n \geq 0\}$.

11.C. (25 PTS.) $L = \left\{0^k w \overline{w} 1^k \;\middle|\; 0 \leq k \leq 3, w \in \{0, 1\}^+\right\}$, where $\overline{w}$ is the complement bit-wise not operator. For $w = w_1 w_2 \ldots w_m \in \{0, 1\}^*$, we define $\overline{w} = \overline{w_1}\,\overline{w_2} \ldots \overline{w_m}$, for $\overline{0} = 1$ and $\overline{1} = 0$.

11.D. (25 PTS.) Suppose L is not regular. Show that $L \cup L'$ is not regular for any finite language L'. Give a simple example to show that $L \cup L'$ may be regular if L' is infinite.

12 (100 PTS.) Context Free Grammar

Describe a context free grammar for the following languages in **12.A.** to **12.C.**. Clearly explain how they work and the role of each non-terminal. Unclear grammars will receive little to no credit.

12.A. (20 PTS.) $L_1 = \left\{ 0^i 1^j 2^k 3^\ell 4^t \;\middle|\; i, j, k, \ell, t \geq 0 \text{ and } i + j + k + t = \ell \right\}$.

12.B. (20 PTS.) $L_2 = \{0, 1\}^* \setminus \{0^n 1^n \mid n \geq 0\}$, i.e., the complement of the language $\{0^n 1^n \mid n \geq 0\}$.

12.C. (20 PTS.) $L_3 = \{0^i 1^j 2^k \mid k = 2(i + j)\}$.

12.D. (40 PTS.) Prove that your grammar for L_3 in **12.C.** is correct. You need to prove that $L_3 \subseteq L(G)$ and $L(G) \subseteq L_3$ where G is your grammar from part **12.C.**. (See solved problem for an example of how this is done.)