1. Let $M = (\Sigma, Q, s, A, \delta)$ be an arbitrary DFA that accepts $L(M)$. Construct and NFA $N$ that accepts the language $\text{half}(L(M)) := \{w \mid ww \in L(M)\}$.

Solution:
We define a new NFA $N = (\Sigma, Q', s', A', \delta')$ with $\varepsilon$-transitions that accepts $\text{half}(L)$, as follows:

- $Q' = (Q \times Q \times Q) \cup \{s'\}$
- $s'$ is an explicit state in $Q'$
- $A' = \{(h, h, q) \mid h \in Q \text{ and } q \in A\}$
- $\delta'(s', \varepsilon) = \{(s, h, h) \mid h \in Q\}$
- $\delta'((p, h, q), a) = \{(\delta(p, a), h, \delta(q, a))\}$

$N$ reads its input string $w$ and simulates $M$ reading the input string $ww$. Specifically, $N$ simultaneously simulates two copies of $M$, one reading the left half of $ww$ starting at the usual start state $s$, and the other reading the right half of $ww$ starting at some intermediate state $h$.

- The new start state $s'$ non-deterministically guesses the “halfway” state $h = \delta^*(s, w)$ without reading any input; this is the only non-determinism in $N$.
- State $(p, h, q)$ means the following:
  - The left copy of $M$ (which started at state $s$) is now in state $p$.
  - The initial guess for the halfway state is $h$.
  - The right copy of $M$ (which started at state $h$) is now in state $q$.
- $N$ accepts if and only if the left copy of $M$ ends at state $h$ (so the initial non-deterministic guess $h = \delta^*(s, w)$ was correct) and the right copy of $M$ ends in an accepting state.

Rubric: 5 points =
+ 1 for a formal, complete, and unambiguous description of a DFA or NFA
  - No points for the rest of the problem if this is missing.
+ 3 for a correct NFA
  - $-1$ for a single mistake in the description (for example a typo)
+ 1 for a brief English justification. We explicitly do not want a formal proof of correctness, but we do want one or two sentences explaining how the NFA works.