7 (100 pts.) Construct NFAs
For each of the following languages over \( \Sigma = \{3, 7, 4\} \), draw an NFA that accepts them. Your NFA should have a small number of states (at most say 14 states). Provide a brief explanation for your solution.

7.A. (20 pts.) \( \Sigma^*3\Sigma^*7\Sigma^*4\Sigma^* \)
7.B. (20 pts.) \( (3(3 + 7))^*3 + 4(3 + 4)^*4 + 7(4 + 7)^*7)^* \)
7.C. (20 pts.) All strings in \( \Sigma^* \) that have a substring in \( 34(3 + 4 + 7)^27 \).
7.D. (20 pts.) All strings in \( \Sigma^* \) that contain the substrings 344 and 443.
7.E. (20 pts.) All strings in \( \Sigma^* \) that satisfy at least one of the following:
   - The number of times 4 appears is divisible by 4.
   - Every non-empty maximal substring of consecutive 7s is odd.
   - Every non-empty maximal substring of consecutive 3s is divisible by 3.

8 (100 pts.) DFAs to NFAs
Given a DFA \( M = (\Sigma, Q, \delta, s, A) \) that accepts \( L \), construct an NFA \( N \) that accepts the following languages. You can assume \( \Sigma = \{0, 1\} \) in 8.A. and 8.B. Provide a brief explanation for your solution.

8.A. (25 pts.) RemoveOnes(\( L \)) := \( \{w \in L \mid 0^\#_0(w) \} \); i.e., removes all 1s from the strings.
8.B. (25 pts.) RemoveOnes\(^{-1}(L) := \{w \in \Sigma^* \mid 0^\#_0(w) \in L\}; i.e., puts back the 1s.
8.C. (25 pts.) Add-k-Ones(\( L \)) := inserts \( k \) 1s into the string. For example, Add-3-Ones(\( L \)) := \( \{x1y1z1w \mid xyzw \in L\} \).
8.D. (25 pts.) Substrings(\( L \)) := \( \{y \mid xyz \in L \text{ for some } x, y, z \in \Sigma^*\} \); i.e., the language of all substrings of strings in \( L \). For example, if \( L = \{ABC\}, \text{Substrings}(L) = \{\epsilon, A, B, C, AB, BC, ABC\} \).

9 (100 pts.) Reg. Exp. to NFA to DFA
For each of the following regular expressions:
1. Construct an NFA corresponding to the regular expression using Thompson’s algorithm.
2. Use the incremental subset construction to convert the NFA to a DFA
3. Describe in natural english text the language defined by the regular expression.
4. Create another DFA with at most say 4 states to recognize the language.

9.A. (50 pts.) \( 1^*(01^*01^*)^*01^* \)
9.B. (50 pts.) \( (10 + 0)^7(1 + \epsilon) \)