HW 3

Due on **Monday, February 10, 2020 at 11pm**

CS/ECE 374 B: Algorithms & Models of Computation, Spring 2020 Version: **2.5**

Submission instructions as in previous homeworks.

7 (100 PTS.) Construct NFAs

For each of the following languages over $\Sigma = \{3, 7, 4\}$, draw an NFA that accepts them. Your NFA should have a small number of states (at most say 14 states). Provide a brief explanation for your solution.

7.A. (20 PTS.) $\Sigma^* 3 \Sigma^* 7 \Sigma^* 4 \Sigma^*$

7.B. (20 PTS.) $\big(3(3+7)^*3 + 4(3+4)^*4 + 7(4+7)^*7\big)^*$

7.C. (20 PTS.) All strings in Σ^* that have a substring in $34(3+4+7)^2 7$.

7.D. (20 PTS.) All strings in Σ^* that contain the substrings 344 and 443.

7.E. (20 PTS.) All strings in Σ^* that satisfy at least one of the following:

- The number of times 4 appears is divisible by 4.
- Every non-empty maximal substring of consecutive 7s is odd.
- Every non-empty maximal substring of consecutive 3s is divisible by 3.

8 (100 PTS.) DFAs to NFAs

Given a DFA $M = (\Sigma, Q, \delta, s, A)$ that accepts L, construct an NFA N that accepts the following languages. You can assume $\Sigma = \{0, 1\}$ in **8.A.** and **8.B.**. Provide a brief explanation for your solution.

8.A. (25 PTS.) $\mathit{RemoveOnes}(L) := \{0^{\#_0(w)} \mid w \in L\}$; i.e., removes all 1s from the strings.

8.B. (25 PTS.) $\mathit{RemoveOnes}^{-1}(L) := \{w \in \Sigma^* \mid 0^{\#_0(w)} \in L\}$; i.e., puts back the 1s.

8.C. (25 PTS.) $\mathit{Add\text{-}k\text{-}Ones}(L)$:= inserts k 1s into the string. For example, $\mathit{Add\text{-}3\text{-}Ones}(L) := \{x1y1z1w \mid xyzw \in L\}$.

8.D. (25 PTS.) $\mathit{Substrings}(L) := \{y \mid xyz \in L \text{ for some } x, y, z \in \Sigma^*\}$; i.e., the language of all substrings of strings in L. For example, if $L = \{ABC\}$, $\mathit{Substrings}(L) = \{\epsilon, A, B, C, AB, BC, ABC\}$.

9 (100 PTS.) Reg. Exp. to NFA to DFA

For each of the following regular expressions:

1. Construct an NFA corresponding to the regular expression using Thompson's algorithm.
2. Use the incremental subset construction to convert the NFA to a DFA
3. Describe in natural english text the language defined by the regular expression.
4. Create another DFA with at most say 4 states to recognize the language.

9.A. (50 PTS.) $1^*\big(01^*01^*\big)^*01^*$

9.B. (50 PTS.) $\big(10 + 0\big)^*(1 + \epsilon)$