HW 2

Due on **Monday, February 3, 2020 at 11pm**

CS/ECE 374 B: Algorithms & Models of Computation, Spring 2020 Version: **1.06**

Submission instructions as in previous homeworks.

4 (100 PTS.) Regular Expressions.

For each of the following languages over the alphabet $\{0, 1\}$, give a regular expression that describes that language, and briefly argue why your expression is correct.

4.A. (20 PTS.) All strings that do not contain the substring 011.

4.B. (20 PTS.) All strings that do not contain the subsequence 011.

4.C. (20 PTS.) All strings that start in 00 and contain 001 as a substring.

4.D. (20 PTS.) All string that contain either the substring 10 or the substring 01, but not both.

4.E. (20 PTS.) All strings in which every nonempty maximal substring of consecutive 0s is of even length. For instance 01100 is not in the language while 10000111001 is.

5 (100 PTS.) DFA

For each of the below languages L, describe a DFA that accepts L. Argue that your machine accepts every string in L and nothing else, by explaining what each state in your DFA *means*.

You may either draw the DFA or describe it formally, but the states Q, the start state s, the accepting states A, and the transition function δ must be clearly specified.

5.A. (50 PTS.) Let L be the set of all strings in $\{0, 1\}^*$ that contain at least two occurences the substrings 100.

5.B. (50 PTS.) Let L be the set of all strings in $\{0, 1, 2\}^*$ that represent ternary numbers divisible by 5 (i.e., numbers in base 3). For example, 120 would be in the language since $120_3 = 1 \cdot 3^2 + 2 \cdot 3 = 15$, while 200 would not. (Hint: It might be easier to describe this DFA than to draw it.)

6 (100 PTS.) More DFAs

(This exercise is about writing things formally – it is not difficult once you have cut through the formalism.)

6.A. (30 PTS.) Let $M = (Q, \Sigma, \delta, s, A)$ be a DFA. A state $q \in Q$ is ***bad***, if for all strings $w \in \Sigma^*$ we have that $\delta^*(q, w) \notin A$. Let $B(M) \subseteq Q$ be the set of bad states of M. Consider the DFA $M' = (Q, \Sigma, \delta, s, B(M))$. What is the language $L(M')$? Prove formally your answer!

6.B. (20 PTS.) Prove that if $x \in L(M')$ and $y \in \Sigma^*$, then $xy \in L(M')$.

6.C. (50 PTS.) Let L_1 and L_2 be two regular languages over Σ accepted by the DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$, and $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$, respectively.
Describe a DFA $M = (Q, \Sigma, \delta, s, A)$ in terms of M_1 and M_2 that accepts

$$L = \{w \mid w \in L_2 \text{ and no prefix of } w \text{ is in } L_1\}$$

Formally specify the components Q, δ, s, and A for M in terms of components of M_1 and M_2.