Instructions: As in previous homeworks.

Problem 2.1: For each of the following languages over the alphabet \{0, 1\}, give a regular expression that describes that language, and briefly argue why your expression is correct.

(a) All strings that contain 10110110 or 1101 as a substring.
(b) All strings that begin with 110 and do not end with 0110.
(c) All strings \(x\) such that the number of 0's in \(x\) is divisible by 3 and \(x\) contains 1101 as a substring.
(d) All strings \(x\) such that between any two 1's in \(x\), the number of 0's is divisible by 3. (For example, 0100010000011100 is in the language, but 01000100000101 is not.)

Problem 2.2: Describe a DFA that accepts each of the following languages over the alphabet \{0, 1\}. Describe briefly what each state in your DFA means.

(a) All strings that contain 101100 as a substring.
(b) All strings \(x\) such that the number of 0's in \(x\) is divisible by 3 and \(x\) does not end in 110.
   [Hint: use the product construction.]
(c) All strings \(x\) such that between any two 1's in \(x\), the number of 0's is divisible by 3. (For example, 0100010000011100 is in the language, but 01000100000101 is not.)

Problem 2.3: Describe a DFA that accepts each of the following languages. Describe briefly what each state in your DFA means. Do not attempt to draw your DFA (the number of states could be huge!). Instead, give a formal description of the states \(Q\), the start state \(s\), the accepting states \(A\), and the transition function \(\delta\). Describe briefly what each state in your DFA means.

(a) All strings in \(\{0, 1, 2\}^*\) such that the number of 0's is divisible by 11, or the number of 1's is divisible by 13, or the number of 2's is divisible by 17.
(b) The language \(L\) from Problem 1.2, i.e., of all strings in \(\{0, 1\}^*\) that contain a balanced substring with length at least 6. (Recall that a string is balanced if it has the same number of 0's and 1's.)
   [Hint: you may use the result from Problem 1.2.]