| April 30, 2019 11:00 AM ||

Final

Mon May 6, 1:30p - 4:30p
(175 min)

cheat sheet as before

7 problems

Undecidability

#11. $L = \{\langle M\rangle \mid$ TM M accepts exactly one string of length ℓ $\forall \ell \geq 0\}$

$= \{\langle M\rangle \mid L(M)$ has exactly one string of - - - $\}$

is undecidable.

Pf: Assume L is decided by alg'm Funny($\langle M\rangle$).

We'll design an alg'm to solve Halting:

Halting($\langle M, w\rangle$):

1. encode the following TM M_w':

$M_w'(x)$:
run M on w
if $x \in 0^*$ return true
else false

2. if Funny($\langle M_w'\rangle$) return true
return true
else return false.

Then $L(M_w') = \begin{cases} 0^* & \text{if M halts on w} \\ \emptyset & \text{else} \end{cases}$

Then $L(M_w') = \begin{cases} 0 & \text{if } M \text{ halts on } \ldots \\ \emptyset & \text{else} \end{cases}$

Halting($\langle M, w\rangle$) returns true
$\iff$ Funny($\langle M_w' \rangle$) returns true
$\iff$ $L(M_w')$ contains exactly 1 string of each length
$\iff$ M halts on w.

Halting is decidable: contra! $\square$

NP-Complete

#3b. Ultra-HC:

Input: undirgraph $G=(V,E)$.
Output: yes iff $\exists$ closed walk that visits every vertex exactly once, except ≤ 1 vertex may be visited more than once.

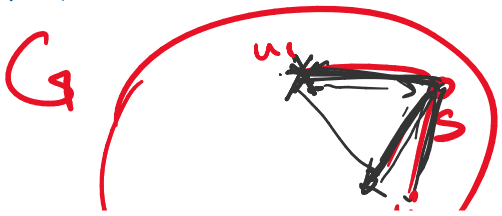
① Ultra-HC $\in$ NP:
certificate: closed walk $\leftarrow$ has length $O(n)$ ← why? polysize
certifier: check C is ultra-Ham cycle
polytime

② HC $\leq_p$ Ultra-HC:

Given input to HC: undir graph $G=(V,E)$,
Construct input to Ultra-HC: new graph $G'=(V',E')$

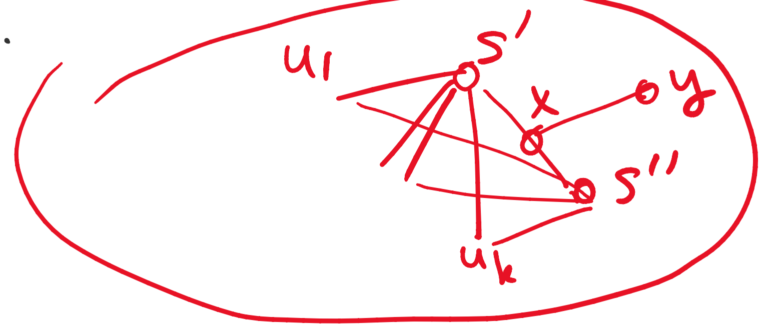
as follows:

(CORRECTED SOL'N!!)



Fix vertex $s \in V$, with neighbors $u_1, \dots, u_k$

G' :



$$V' = V - \{s\} \cup \{s', s'', x, y\}$$

$$E' = E - \{su_i : i = 1,..,k\} \cup \{s'u_i : i = 1,..,k\} \cup \{s''u_i : i = 1,..,k\} \cup \{s'x, xs'', xy\}$$

Construction $G \to G'$ takes polytime.

Correctness: $\exists$ Ham cycle C in G $\iff$ $\exists$ ultra Ham cycle C' in G'

Pf: ($\Rightarrow$) Given $C = v_1 v_2 \cdots v_n v_1$,
wlog say $v_1 = s$.
Let $C' = s' v_2 \cdots v_n s'' x y x s'$
is an ultra Ham cycle
(only x is repeated)

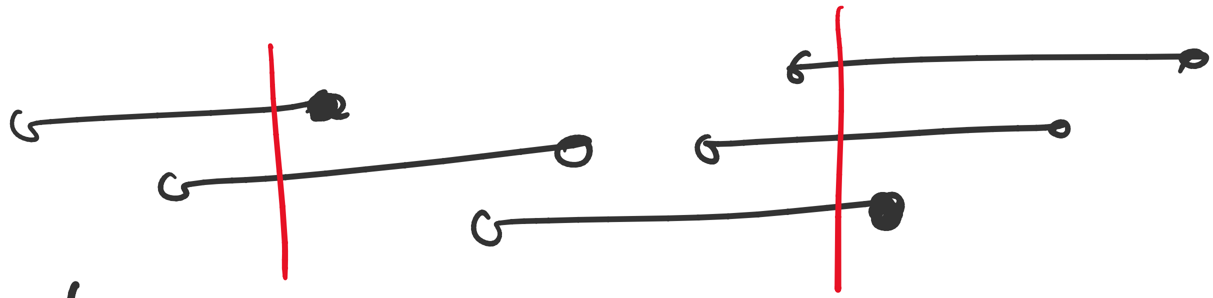
($\Leftarrow$) Given ultra Ham cycle C',
x must be repeated
(since it's the only way to get to y)
So all other vertices are visited once
(including s', s'')
So C' contains a path from s' to s''
visiting all vertices in $V - \{s\}$
once
This maps back to a Ham cycle in G. $\square$

(**Note:** may assume NP-completeness of
3SAT, indep set, vertex cover,
clique, 3-coloring, Ham cycle/path,
subset-sum
remember defn's of these problems!)

Greedy

#19 (mid2 fall '16)

Given set of n intervals
$[a_1, b_1], \ldots, [a_n, b_n]$,
find smallest set P of pts that stabs X.



$P = \emptyset$

repeat {

  pick i with smallest b_i

  insert b_i to P

  remove all intervals stabbed by b_i

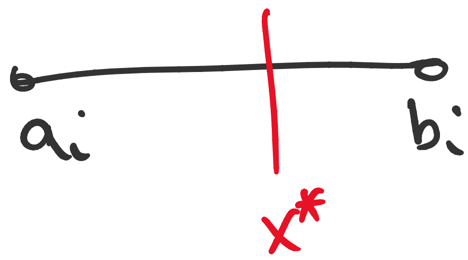
}

Correctness Pf:

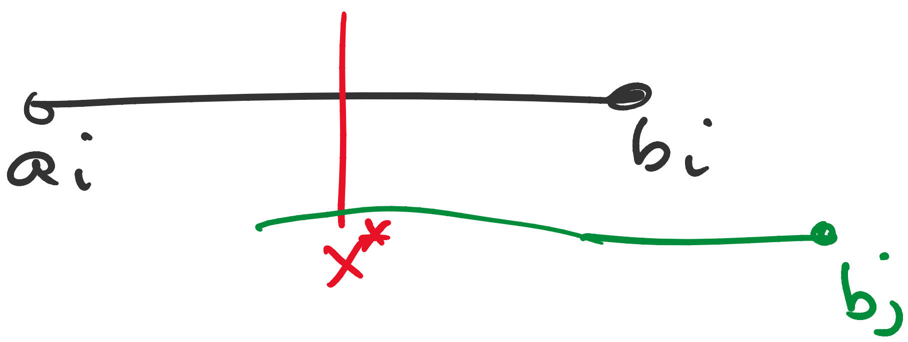
Let P^* be opt sol'n.

Let i minimizes b_i.

Let $x^* \in P^*$ that stabs (a_i, b_i)



For every interval $(a_j, b_j]$ stabbed by x^*,



Know $b_i \leq b_j$.

$\Rightarrow$ $[a_j, b_j]$ is stabbed by b_i

$\Rightarrow$ $P^* - \{x^*\} \cup \{b_i\}$ is a feasible sol'n

$\Rightarrow$ $P - \{x^*\} \cup \{b_i\}$ is a feasible sol'n

& is also optimial

& uses b_i.

Repeat arg.

$\Rightarrow$ $\exists$ opt sol'n that agrees with greedy.

$\square$