no discussion sessions tomorrow

Dynamic Prog. (Cont'd)

Last Ex: Parsing

Given CFG $G = (V, T, P, S)$ & String $x = a_1a_2\ldots a_n \in T^*$, decide whether $x \in L(G)$

Note: may assume that all productions in $P$ are of the form

- $A \rightarrow BC$ $(A, B, C \in V)$
- $A \rightarrow a$ $(A \in V, a \in T)$

Chomsky Normal Form (CNF)

E.g. $S \rightarrow AS | AB$
$A \rightarrow BC$
$B \rightarrow SA | AC | 1$
$C \rightarrow AB | CC | 0$

(e.g. $A \rightarrow BCDE$ can be converted to $A \rightarrow XY, X \rightarrow BC, Y \rightarrow DE$)

CYK Alg'm (Cocke-Younger-Kasami '70)

Define subproblems: $(i \leq j \leq n, A \in V)$
Define subproblems: \((1 \leq i \leq j \leq n, A \in V)\)

\(f(i,j,A) = \text{true iff} \)

-the substring \(a_i a_{i+1} \ldots a_j\)

-can be generated by \(A\).

Final Ans: \(f(1,n,S)\)

Base cases: \(f(i,i,A) = \text{true iff} \)

\(A \rightarrow a_i\) is in \(P\).

Recursive formula:

\[
f(i,j,A) = \bigvee_{\substack{A \rightarrow BC \\ A \in P \land B \in P \land C \in P}} \bigvee_{k=i}^{j-1} (f(i,k,B) \land f(k+1,j,C))
\]

Evaluation order:

-increased order of \(j-i\)

Runtime:

\# subproblems = \(O(n^2)\)

time per subproblems = \(O(|P|n)\)

\[\Rightarrow \ O(|V| |P| n^3) \text{ total time}\]
more carefully, \( O(1p^n^3) \) time

( Since each production \( A \rightarrow BC \) is considered once )

Rank: can be improved to \( O(1p^n^{2.373}) \)

(Valliant '75)

for most PL, there are faster linear-time alg's...

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**Graph Alg's**

**Def** A graph \( G \) is \( (V,E) \),

\( V \) = set of vertices

\( E \) = set of edges of form \( (u,v) \) or \( uv \)

if directed

\( n = |V| \)

\( m = |E| \)

if undirected

( or \( \{u,v\} \) )

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**Basic concepts:** adjacency, incidence, paths, connectedness, cycles, trees, ...

**Apps:** road networks, social networks,
(appls: road networks, social networks, web page links, ...)

**Note:** \( n - 1 \leq m \leq n^2 \)

If connected

**Representation**

- **adjacency matrix**
  
  \[
  A[u, v] = \begin{cases} 
  1 & \text{if } (u, v) \in E \\
  0 & \text{else}
  \end{cases}
  \]

  \( \Theta(n^2) \) space (good for dense graphs when \( m \) close to \( n^2 \))

- **adjacency lists**
  
  for each \( u \in V \),
  
  Store linked list \( \text{Adj}[u] = \{ v : (u, v) \in E \} \)

  ![Diagram of adjacency list]

  Space \( O(n + \sum_{u \in V} |\text{Adj}[u]|) \)

  \[
  = O(n + \sum_{u \in V} \text{out-deg}(u))
  \]

  \[
  = O(n + m)
  \]

  (good for sparse graphs when \( m \) close to \( n \))

**Basic Problems:**

Is there a path from \( s \) to \( t \)?
- Is there a path from s to t?
- Find all vertices reachable from s?
- Is graph connected?  \{ directed? \}
- Find all connected components