February 14, 2019 11:00 AM

Midterm 1

Feb 18 Mon 7pm - 9pm

reg exprs, DFA, NFA, CFG (up to lec 7)
but no TM

see midterm cover page (from website)
allow cheat sheet
- handwritten, double-sided

see past midterms (e.g. Fall '17) (links from piazza)

#43. all strings with equal # of "01" & "10"

0110100 1

000111100000111111110000 yes

0011110001111 no

Case 1. if start with block of 0, end with block of 0

Case 2. . . . . 1, ... 1

regular

$$0^+(1^+0^+)^* + 1^+(0^+1^+)^*$$

Case 1 Case 2

$+\varepsilon$

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72. If L is regular, prove $\text{OnlyOnes}(L)$ is regular
$= \{ 1^{\#_1(w)} \mid w \in L \}$.

e.g. $01101100 \in L \Rightarrow \underline{1111} \in \text{OnlyOnes}(L)$

Given DFA for L
$M = (Q, \Sigma, \delta, s, A)$,

Construct ~~DFA~~/NFA for $\text{OnlyOnes}(L)$:
$M' = (Q', \Sigma, \delta', s', A')$

idea. as M' reads ~~in~~ 1, feed 1 to M
at any time, feed 0 to M

$$\delta'(q, 1) = \{\delta(q, 1)\}$$

$$\delta'(q, \varepsilon) = \{\delta(q, 0)\}$$

$$Q' = Q$$
$$s' = s$$
$$A' = A$$

Claim: $\delta'^*(q, 1^i) = \{ \delta^*(q, w) \mid w \in \{0,1\}^*, \#_1(w) = i \}$

Given reg expr r_1 for L_1, r_2 for L_2, OR

Given reg expr r_1, r_2, for L_2
r_1' for OnlyOnes(L_1),
r_2' for OnlyOnes(L_2),

OnlyOnes$(L_1 L_2)$: $r_1' \cdot r_2'$
OnlyOnes(L_1^*) : r_1^*

OnlyOnes(ε) : ε
OnlyOnes$(\{1\})$: $\{1\}$
OnlyOnes$(\{0\})$: ε

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103. $\{0^m 1^n \mid m \leq 2n \ \& \ n \leq 2m\}$

$S \to A \mid B$

$A \to 00A1 \mid C$

$B \to \underline{0B11} \mid C$

$C \to 0C1 \mid \varepsilon$

$L(C) = \{0^i 1^i \mid i \geq 0\}$

pf by induction

$L(B) = \{0^j 0^i 1^i 1^{2j} \mid i,j \geq 0\}$

$= \{0^{i+j} 1^{i+2j} \mid i,j \geq 0\}$

pf by induction.

$L(A) = \{0^{2k} 0^i 1^i 1^k \mid i,k \geq 0\}$

$= \{0^{i+2k} 1^{i+k} \mid i,k \geq 0\}$

$= \{ 0^{i+2k} 1^{i+k} \mid i, k \geq 0 \}$

symmetric

$L(S) = L(A) \cup L(B)$

$= \{ 0^m 1^n \mid m = i+j,\ n = i+2j$ for some $i, j \geq 0$

or $m = i+2k,\ n = i+k$ for some $i, k \geq 0 \}$

$\|$

$\{ 0^m 1^n \mid m \leq 2n \ \& \ n \leq 2m \}$

$\subseteq$: if $m = i+j$, $n = i+2j$,

$i+j \leq 2(i+2j)$ ✓

$i+2j \leq 2(i+j)$ ✓

if $m = i+2k$, $n = i+k$,

$i+2k \leq 2(i+k)$ ✓

$i+k \leq 2(i+2k)$ ✓

$\supseteq$: if $m \leq 2n$, & $n \leq 2m$,

w.l.o.g. $m \geq n$.

Know $n \leq m \leq 2n$.

Let $k = m-n \geq 0$.

$i = n-(m-n) = 2n-m \geq 0$

$\rightarrow i+2k = 2n-m+2(m-n) = m$

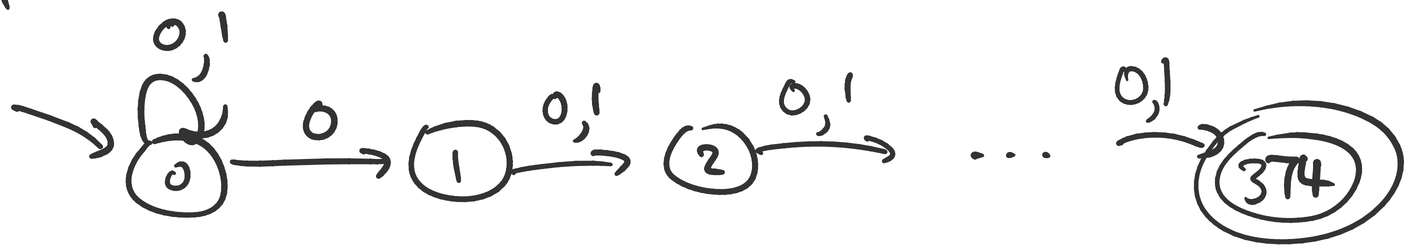
$\rightarrow i+k = n$ ✓

$\square$

66. all strings where 374^{th} symbol from end

66. all strings where 374^{th} symbol from end is 0.

NFA:



$$\begin{cases} \delta(i,0) = \{i+1\} & i = 1,..,374 \\ \delta(i,1) = \{i+1\} \\ \delta(0,0) = \{0,1\} \\ \delta(0,1) = \{0\} \end{cases}$$

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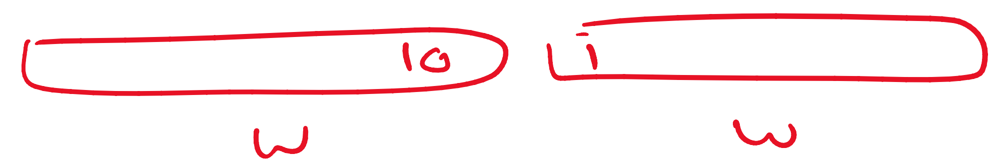
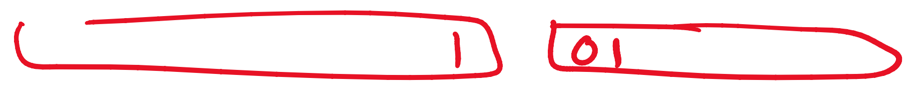
65 H. $\{ w \mid ww \text{ does not contain } 101 \}$

1. w does not contain 101

& 2. not (w ends in 10 & begins in 1)





& 3. not (w ends in 1 & begins 01)

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58. $L = \{ xy \mid \#_0(x) = \#_1(y) \text{ \& } \#_1(x) = \#_0(y) \}$ not regular

$\Downarrow$

$|x| = |y|$

$\Downarrow$
$|x| = |y|$

Pick
$\Rightarrow F = \{0^i \mid i \geq 1\}$.

Consider $u, v \in F$

$u = 0^i$, $v = 0^j$, for $i \neq j$

w.l.o.g. $i < j$.

$\Rightarrow$ Pick $w = 1^i$

Then $uw = 0^i 1^i \in L$

$uv = \underline{0^j 1^i} \notin L$.

$$\underbrace{0000}_{\#_1(x)}\, 1 \,\Big|\, \underbrace{11111}_{\#_0(y)} \qquad \#_1(x) \neq \#_0(y)$$

F is infinite fooling set. $\square$

Another way! by closure prop.

$\underline{L \cap 0^+ 1^+} = \underline{\underline{\{0^i 1^i \mid i \geq 1\}}}$

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28. All strings w s.t.
w^R in binary is divisible by 5.

DFA :

Start w. $\{ w :$ w in binary is div. by $5\}$

101 5

101   5

1010   10

1111   15

remember $w \bmod 5 = i$

$w0 \quad i \to 2i \bmod 5$

$w1 \quad i \to (2i+1) \bmod 5$

$$\begin{cases} \delta(i,0) = 2i \bmod 5 \\ \delta(i,1) = (2i+1) \bmod 5, \end{cases}$$

$i = 0,1,2,3,4$

$A = \{0\}$

$s = 0.$

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w has length j

$w0 \longrightarrow \quad i \to i \bmod 5$

$w1 \longrightarrow \quad i \to i + 2^j \bmod 5$

$2^j \bmod 5 := 2^{j \bmod 4} \bmod 5$

$2^0 \quad 2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5$

$\underbrace{1 \quad 2 \quad 4 \quad 3} \quad \underbrace{1 \quad 2 \quad 4 \quad 3} \ldots$

$i = 0, \ldots, 4, \quad j = 0, \ldots, 3$

$$\left\{\begin{array}{l} \delta((i,j),0) = (i, (j+1) \bmod 4) \\ \delta((i,j),1) = ((i+2^j) \bmod 5, (j+1) \bmod 4). \end{array}\right.$$

20 states