January 29, 2019 11:01 AM

- discussion sessions on this Wed cancelled (due to weather)
- HW2 due 10am Wed

- Regular exprs/langs
- DFA
- NFA

} all equiv.

Thm If L is regular, then L is accepted by some NFA.

Pf: By recursion. $\square$

NFA $\to$ DFA

Thm If L is accepted by NFA M, then L is accepted by some DFA M'.

Pf: idea - remember a subset of states

Given NFA $M = (Q, \Sigma, \delta, s, A)$
$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \underline{\mathcal{P}}(Q)$,

construct DFA $M' = (Q', \Sigma, \delta', s', A')$
$\delta' : \underline{Q'} \times \Sigma \to Q'$:

$Q' = \mathcal{P}(Q)$ (power-set construction or subset construction)

$s' = \varepsilon\text{-reach}(s)$

$A' = \{ S \in Q' \mid S \cap A \neq \emptyset \}$

$$A' = \{ S \in Q' \mid S \cap A \neq \emptyset \}$$

$$\delta'(S, a) = \bigcup_{q \in S} \delta^*(q, a)$$

$$(S \in Q',\ a \in \Sigma)$$

Lemma: $\delta'^*(S, x) = \bigcup_{q \in S} \delta^*(q, x)$

$(S \in Q',\ x \in \Sigma^*)$

Pf: By boring induction...

Then $x \in L(M') \overset{\text{accept in DFA}}{\iff} \delta'^*(s', x) \in A'$

$$\iff \underbrace{\bigcup_{q \in \varepsilon\text{-reach}(s)}}\ \delta^*(q, x) \in A'$$

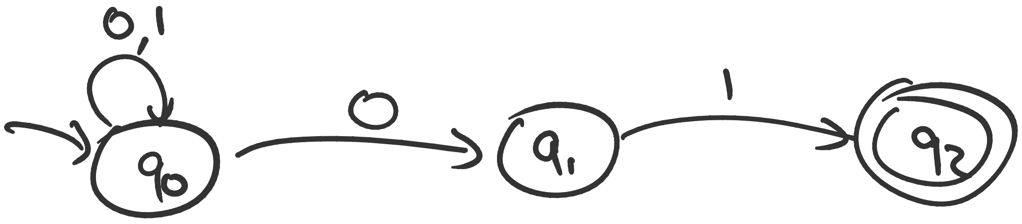
$$\iff \delta^*(s, x) \in A'$$

$$\iff \delta^*(s, x) \cap A \neq \emptyset$$

$$\underset{\text{accept. in NFA}}{\iff} x \in L(M). \quad \square$$

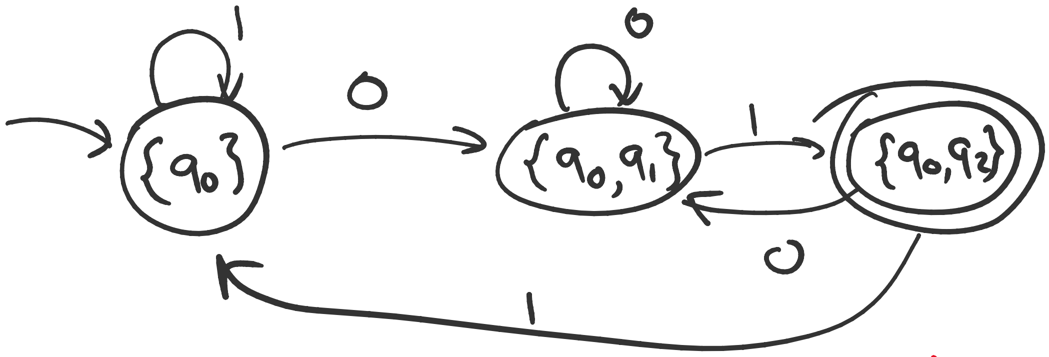
Ex 1

NFA



$\downarrow$

DFA

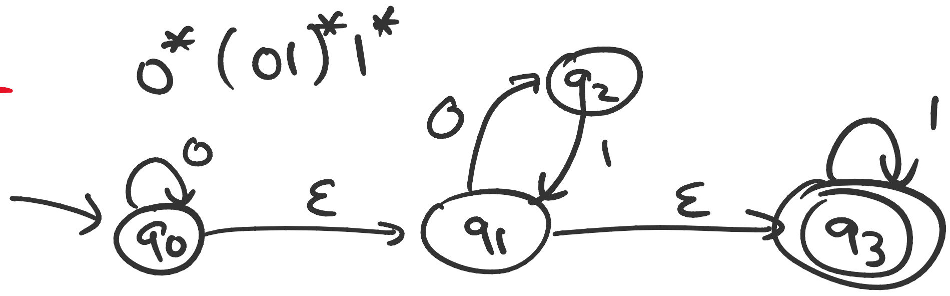


$(\delta(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$
$= \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\})$

$(\delta(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$
$= \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$

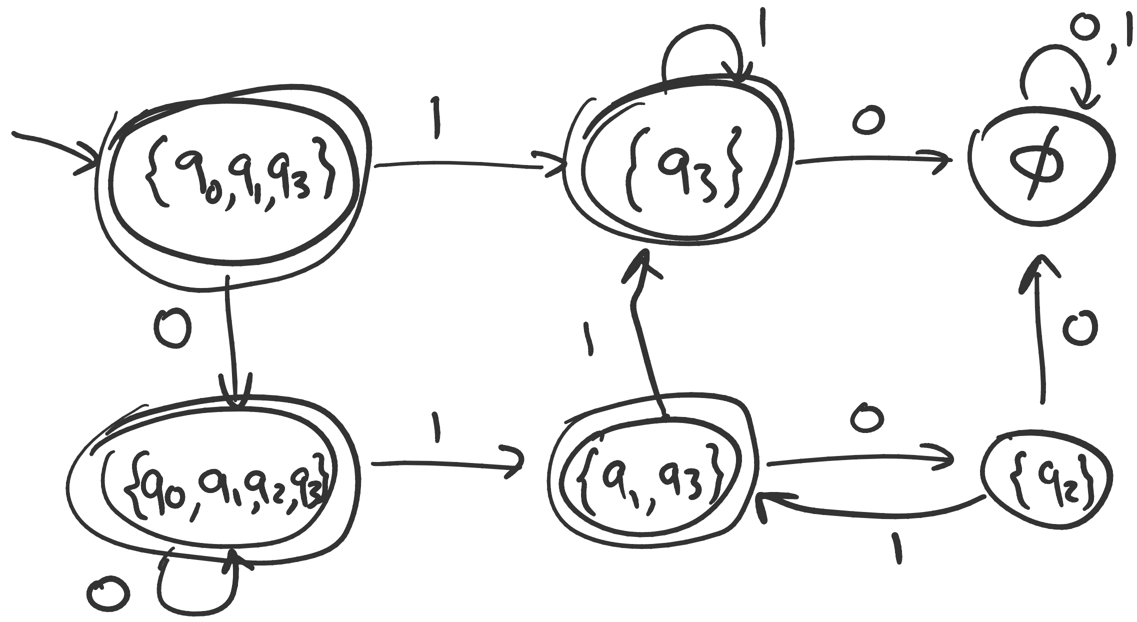
Ex2 $\quad 0^*(01)^*1^*$

NFA



⇓

DFA

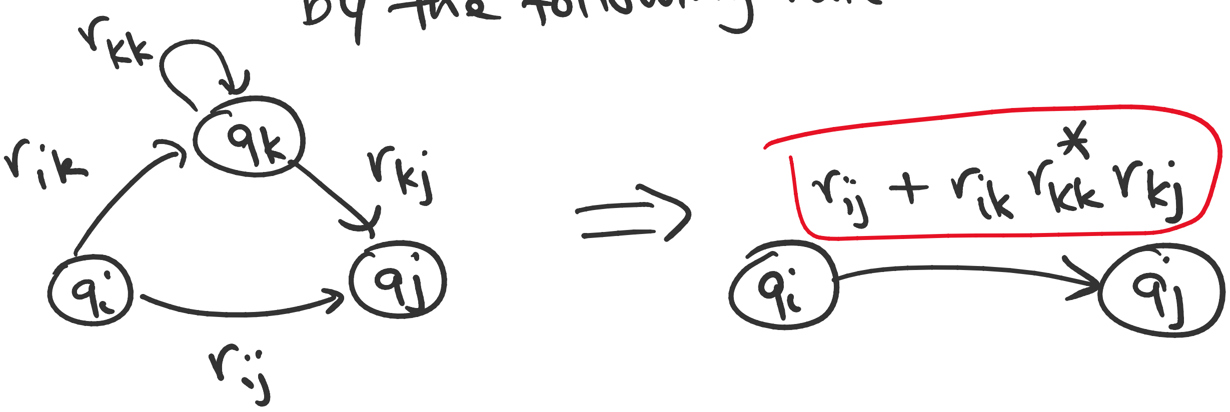


DFA → Regular

Thm If L is accepted by DFA (or NFA) M, then L is regular.

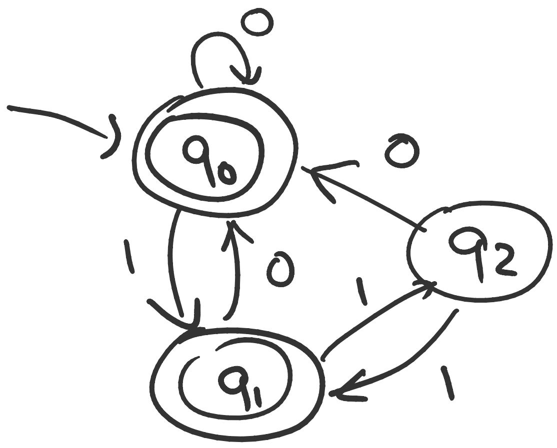
Pf Sketch: Given $M = (Q, \Sigma, \delta, s, A)$, (state elimination method)

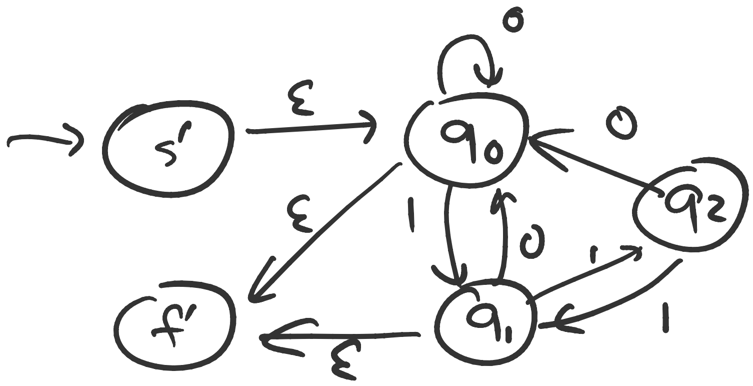
1. add new state s', f' with ε-transitions from s' to s & from A to f'.
2. for each $q_k \in Q$, remove q_k & relabel transitions by the following rule:



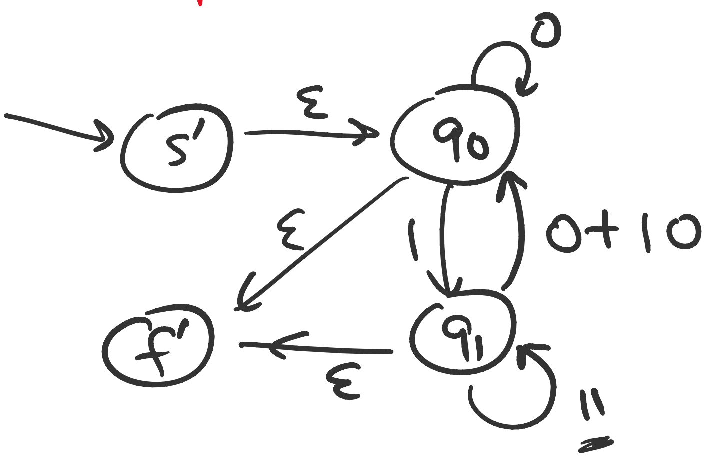
3. return label from s' to f'

Ex

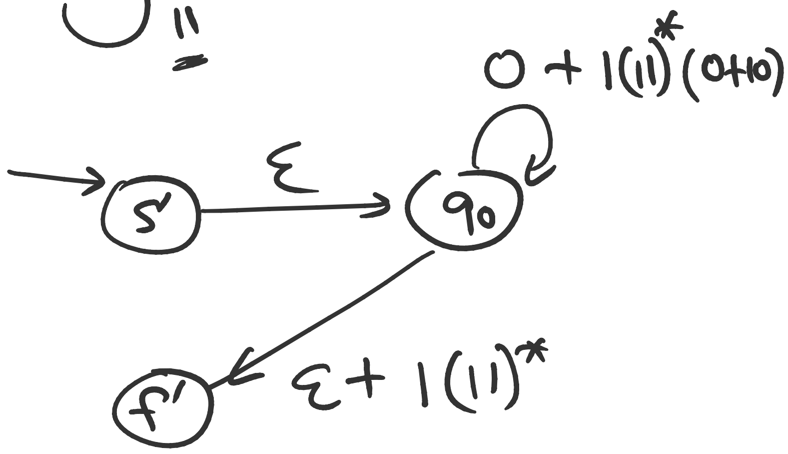




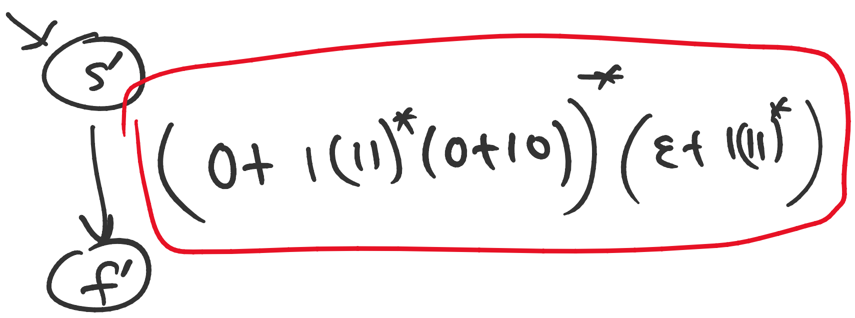
remove q_2



remove q_1



remove q_0



Kleene's Thm (1956)

Kleene's Thm (1956)

L is regular iff L is accepted by some DFA.

$$\begin{tikzcd}
\text{regular} \arrow[r, "\text{recursion}"] & \text{NFA} \arrow[r, "\text{Power-set construct}"] & \text{DFA} \arrow[ll, bend left, "\text{state elimin.}"]
\end{tikzcd}$$

Cor If L is regular, then so is $\overline{L}$.
If L_1, L_2 are regular,
then so is $L_1 \cap L_2$.

Rmk: closed under other ops:
reverse, homomorphism, ...
prefix, suffix, subseq, superseq, ...