In lecture, somebody described an algorithm of Karatsuba that multiplies two \( n \)-digit integers using \( O(n^{\lg 3}) \) single-digit additions, subtractions, and multiplications. In this lab we will look at some extensions and applications of this algorithm.

1. Describe an algorithm to compute the product of an \( n \)-digit number and an \( m \)-digit number, where \( m < n \), in \( O(m^{\lg 3-1} n) \) time.

2. Describe an algorithm to compute the decimal representation of \( 2^n \) in \( O(n^{\lg 3}) \) time. (The standard algorithm that computes one digit at a time requires \( \Theta(n^2) \) time.)

3. Describe a divide-and-conquer algorithm to compute the decimal representation of an arbitrary \( n \)-bit binary number in \( O(n^{\lg 3}) \) time. (Hint: Let \( x = a \cdot 2^{n/2} + b \). Watch out for an extra log factor in the running time.)

Think about later:

4. Suppose we can multiply two \( n \)-digit numbers in \( O(M(n)) \) time. Describe an algorithm to compute the decimal representation of an arbitrary \( n \)-bit binary number in \( O(M(n) \log n) \) time.