Design Turing machines $M = (Q, \Sigma, \Gamma, \delta, \text{start}, \text{accept}, \text{reject})$ for each of the following tasks, either by listing the states $Q$, the tape alphabet $\Gamma$, and the transition function $\delta$ (in a table), or by drawing the corresponding labeled graph.

Each of these machines uses the input alphabet $\Sigma = \{1, \#\}$; the tape alphabet $\Gamma$ can be any superset of $\{1, \#, \text{□}, \text{◵}\}$ where □ is the blank symbol and ◄ is a special symbol marking the left end of the tape. Each machine should reject any input not in the form specified below.

1. On input $1^n$, for any non-negative integer $n$, write $1^n \#1^n$ on the tape and accept.

2. On input $\#^n1^m$, for any non-negative integers $m$ and $n$, write $1^m$ on the tape and accept. In other words, delete all the #s and shift the 1s to the start of the tape.

3. On input $\#1^n$, for any non-negative integer $n$, write $\#1^{2n}$ on the tape and accept. (Hint: Modify the Turing machine from problem 1.)

4. On input $1^n$, for any non-negative integer $n$, write $1^{2n}$ on the tape and accept. (Hint: Use the three previous Turing machines as subroutines.)