Prove that each of the following languages is not regular.

1. \( \{0^{2^n} \mid n \geq 0\} \).
2. \( \{0^{2^n}1^n \mid n \geq 0\} \).
3. \( \{0^{m}1^n \mid m \neq 2n\} \).
4. Strings over \( \{0,1\} \) where the number of 0s is exactly twice the number of 1s.
5. Strings of properly nested parentheses (), brackets [], and braces {}. For example, the string ([]){} is in this language, but the string ([)]{} is not, because the left and right delimiters don’t match.
6. Strings of the form \( w_1\#w_2\#\cdots\#w_n \) for some \( n \geq 2 \), where each substring \( w_i \) is a string in \( \{0,1\}^* \), and some pair of substrings \( w_i \) and \( w_j \) are equal.

Extra problems

7. \( \{0^{n^2} \mid n \geq 0\} \).
8. \( \{w \in (0 + 1)^* \mid w \text{ is the binary representation of a perfect square}\} \).