Designing DFAs via product construction and designing NFAs.

1 Describe a DFA that accepts the following language over the alphabet $\Sigma = \{0, 1\}$.

All strings in which the number of 0s is even and the number of 1s is *not* divisible by 3.

2 All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

For example, the string 1100 is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

3 Design an NFA for the language $(01)^+ + (010)^+$.

Work on these later:

Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{0, 1\}$. You may find it easier to describe these DFAs formally than to draw pictures.

4 All strings w such that $\binom{|w|}{2} \bmod 6 = 4$. $\big($**Hint:** Maintain both $\binom{|w|}{2} \bmod 6$ and $|w| \bmod 6.\big)$

5 (**Hard.**) All strings w such that $F_{\#(10,w)} \bmod 10 = 4$, where $\#(10, w)$ denotes the number of times 10 appears as a substring of w, and F_n is the nth Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$