
Submission instructions as in previous [homeworks](#).

31 (100 PTS.) **OLD Homework problem (not for submission):**

Walk with me

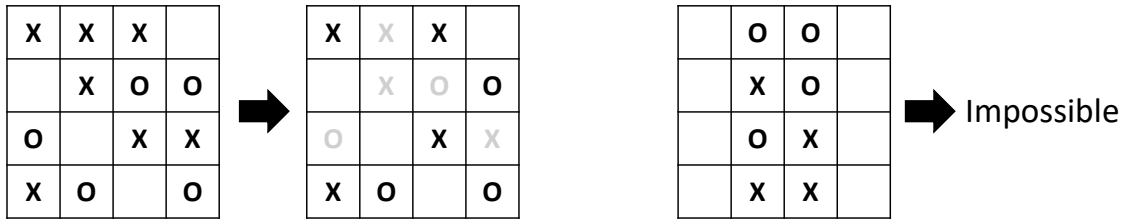
- 31.A.** (50 PTS.) We are given a *directed* graph G with n vertices and m edges ($m \geq n$), where each vertex v has a height $h(v)$. The *cost* of traversing an edge (u, v) is $c(u, v) = |h(v) - h(u)|$. The cost of a walk in G is the sum of the costs of edges in the walk. Prove that finding the minimum cost walk that visits all the vertices on G is **NP-HARD**. (In a walk, vertices and edges may be repeated, and the start and end vertices may be different.)
- 31.B.** (50 PTS.) We are given a directed graph G with n vertices and m edges ($m \geq n$), where each edge e has a set of colors $C(e) \subseteq \{1, \dots, k\}$. Prove that deciding whether there exists a walk that uses all k colors (i.e. the union of the sets of colors of the edges of walk covers all colors.) is **NP-HARD**. (Hint: Reduce from Set Cover.)

32 (100 PTS.) **OLD Homework problem (not for submission):**

Things are hard.

- 32.A.** (20 PTS.) Suppose we have n prisoners P_1, \dots, P_n that we want to place in some disconnected blocks of a prison. Each prisoner is assigned to one block, and he/she will not be able to access other blocks. However, some prisoners are bitter enemies (going all the way back to kindergarten) and cannot be placed in the same block. Given integers n and k and a list of enemies for each of the n prisoners, we want to determine whether k blocks are sufficient to house all the prisoners? Prove that this problem is **NP-HARD**. You can safely assume that every block has unbounded capacity.
- 32.B.** (40 PTS.) Let G be an arbitrary directed weighted graph with n vertices and m edges such that no edge weight is zero (weights can be positive or negative). Prove that finding a *zero-length* (i.e., zero weight) Hamiltonian cycle in G is **NP-COMplete**.
- 32.C.** (40 PTS.) Consider the following **XO** puzzle. You are given an $n \times m$ grid of squares where each square has an **X**, an **O** or is empty. Your goal is to erase some of the **Xs** and **O**s so that
- every row contains at least one **X** or one **O**, and
 - no column contains both **X** and **O**.

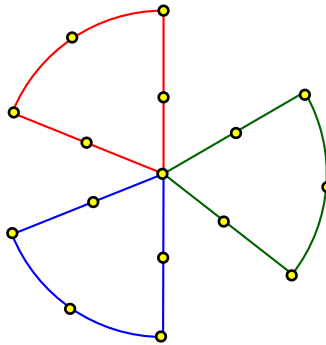
For some input grids, it is impossible to solve the puzzle. The figure below shows two examples: a grid that is solvable and a grid that is impossible to solve. Prove that, given a grid, it is **NP-HARD** to determine whether the puzzle is solvable. (Hint: Reduce from 3SAT.)



33 (100 PTS.) OLD Homework problem (not for submission):

Fan, fan, fan.

An undirected graph is a *3-blade-fan* if it consists of three cycles $C_1, C_2,$ and C_3 of k nodes each and they all share exactly one node. Hence, the graph has $3k - 2$ nodes. The figure below shows a *3-blade-fan* of 16 nodes.



Given an undirected graph G with n vertices and m edges and an integer k , the FAN problem asks whether or not there exists a subgraph of G which is a *3-blade-fan*. Prove that FAN is NP-COMPLETE.