Any dynamic programming solution should be done using an iterative algorithm.

25 (100 pts.) Rainbow walk

We are given a directed graph with $n$ vertices and $m$ edges ($m \geq n$), where each edge $e$ has a color $c(e)$ from $\{1, \ldots, k\}$.

25.A. (20 pts.) Describe an algorithm, as fast as possible, to decide whether there exists a closed walk that uses all $k$ colors. (In a walk, vertices and edges may be repeated. In a closed walk, we start and end at the same vertex.)

25.B. (80 pts.) Now, assume that there are only 3 colors, i.e., $k = 3$. Describe an algorithm, as fast as possible, to decide whether there exists a walk that uses all 3 colors. (The start and end vertex may be different.)

26 (100 pts.) Stay safe

We are given an undirected graph with $n$ vertices and $m$ edges ($m \geq n$), where each edge $e$ has a positive real weight $w(e)$, and each vertex is marked as either “safe” or “dangerous”.

26.A. (35 pts.) Given safe vertices $s$ and $t$, describe an $O(m)$-time algorithm to find a path from $s$ to $t$ that passes through the smallest number of dangerous vertices.

26.B. (65 pts.) Given safe vertices $s$ and $t$ and a value $W$, describe an algorithm, as fast as possible, to find a path from $s$ to $t$ that passes through the smallest number of dangerous vertices, subject to the constraint that the total weight of the path is at most $W$.

27 (100 pts.) Stay stable

We are given a directed graph with $n$ vertices and $m$ edges ($m \geq n$), where each edge $e$ has a weight $w(e)$ (you can assume that no two edges have the same weight). For a cycle $C$ with edge sequence $e_1e_2\cdots e_\ell e_1$, define the fluctuation of $C$ to be

$$f(C) = |w(e_1) - w(e_2)| + |w(e_2) - w(e_3)| + \cdots + |w(e_\ell) - w(e_1)|.$$

27.A. (10 pts.) Show that the cycle with the minimum fluctuation cannot have repeated vertices or edges, i.e., it must be a simple cycle.

27.B. (90 pts.) Describe a polynomial-time algorithm, as fast as possible, to find the cycle with the minimum fluctuation.