HW 6

Due on **Wednesday, October 14, 2020 at 10am**

CS/ECE 374: Algorithms & Models of Computation, Fall 2020 Version: **1.1**

Submission instructions as in previous homeworks.

16 (100 PTS.) OLD Homework problem (not for submission):

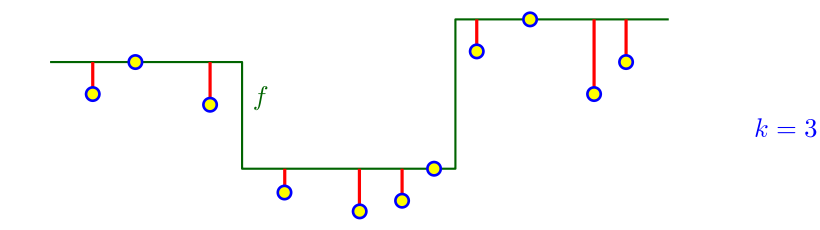
Simplifying data.

A *k-step function* is a function of the form

$$f(x) = b_i \qquad \text{if } a_i \le x < a_{i+1} \;\; (i = 0, \ldots, k-1)$$

for some $-\infty = a_0 < a_1 < \cdots < a_{k-1} < a_k = \infty$ and some $b_0, b_1, \ldots, b_{k-1}$.

We are given n data points $p_1 = (x_1, y_1), \ldots, p_n = (x_n, y_n)$ and a number k between 1 and n. Our objective is to find a k-step function f such that $f(x_i) \ge y_i$ for all $i \in \{1, \ldots, n\}$, while minimizing the total "error" $\sum_{i=1}^{n} (f(x_i) - y_i)$ (this is the total length of the red vertical segments in the figure below).



16.A. (70 PTS.) Describe an algorithm, as fast as possible, that computes the minimum total error of the optimal k-step function. Bound the running time of your algorithm as a function of n and k.

[Note: in dynamic programming questions such as this, first give a clear English description of the function you are trying to evaluate, and how to call your function to get the final answer, then provide a recursive formula for evaluating the function (including base cases). If a correct evaluation order is specified clearly, iterative pseudocode is not required.]

16.B. (30 PTS.) Describe how to modify your algorithm in (A) so that it computes the optimal k-step function itself.

17 (100 PTS.) OLD Homework problem (not for submission):

Closest subsequence

Define the L_1-*distance* between two sequences of real numbers $\langle a_1, \ldots, a_m \rangle$ and $\langle b_1, \ldots, b_m \rangle$ to be $|a_1 - b_1| + \cdots + |a_m - b_m|$.

Consider the following problem: given two sequences of real numbers $A = \langle a_1, \ldots, a_m \rangle$ and $B = \langle b_1, \ldots, b_n \rangle$ with $m \le n$, find a subsequence of B of length m that minimizes its L_1-distance to A.

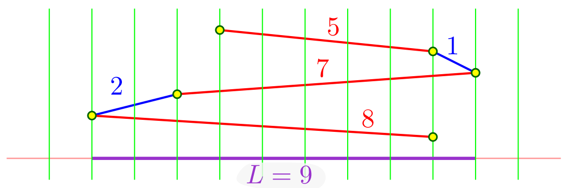
17.A. (70 PTS.) Describe an algorithm, as fast as possible, that computes the L_1-distance of the optimal subsequence of B to A. Bound the running time of your algorithm as a function of m and n.

17.B. (30 PTS.) Describe how to modify your algorithm in (A) so that it computes the optimal subsequence itself.

18 (100 PTS.) **OLD Homework problem (not for submission):**
Fold it!

We are given a "chain" with n links of lengths $a_1, \ldots, a_n$, where each a_i is a positive integer. We are also given a positive integer L. We want to determine if it is possible to "fold" the chain (in one dimension) so that the length of the folded chain is at most L. More formally, we want to decide whether there exists $t \in [0, L]$ and $s_1, \ldots, s_n \in \{-1, +1\}$ such that $t + \sum_{i=1}^{j} s_i a_i \in [0, L]$ for all $j \in \{0, \ldots, n\}$. (Here, t denotes the starting position, and $s_i = \pm 1$ depending on whether we turn rightward or leftward for the ith link.)

Example: for $a_1 = 5$, $a_2 = 1$, $a_3 = 7$, $a_4 = 2$, $a_5 = 8$, and $L = 9$, a solution is shown below.



18.A. (70 PTS.) Provide an $O(nL)$-time algorithm to decide whether a solution exists. (Argue why the stated running time is correct.)

Partial credit would be given to slower solutions with running time $O(nL^2)$ or $O(nL^3)$.

18.B. (30 PTS.) Using (A) as a subroutine, describe an algorithm (as fast as possible) to find the minimum length L such that a valid folding exists. L^* of the best folding. What is the running time of your algorithm as a function of n and L^*?