HW 4

Due on **Wednesday, February 13, 2019 at 10am**

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.01**

Submission instructions as in previous homeworks.

10 (100 PTS.) Aberrant.

10.A. (25 PTS.) Prove that the following language is not regular by providing a fooling set. You need to prove an infinite fooling set and also prove that it is a valid fooling set. For $\Sigma = \{a, b\}$, the language is

$$L = \left\{ ww \;\middle|\; w \in \Sigma^+ \right\}.$$

10.B. (25 PTS.) Same as (A) for the following language. Recall that a ***run*** in a string is a maximal non-empty substring of identical symbols. Let L be the set of all strings in Σ^* that contains two distinct runs of equal length. A few examples about L:

- L contains any string of the form $b^i a^+ b^+ a^i$.
- L contains any string of the form $b^i a^+ b^i$.
- L does not contain the strings $abbaaa$, $abbaaabbbb$.

10.C. (25 PTS.) Suppose you are given two languages L, L' that are not regular but such that $L' \setminus L$ is regular. Prove that $L \cup L'$ is not regular. (Hint: Use closure properties of regular languages.)

10.D. (15 PTS.) Provide a counter-example for the following claim:
Claim: Consider two languages L and L'. If $\overline{L}$ is not regular, L' is regular, and $L \cup L'$ is regular, then $L \cap L'$ is regular.

10.E. (10 PTS.) (Slightly harder[1]) Same as (A) for $L = \left\{ 0^{n^4} \;\middle|\; n \geq 3 \right\}$.

11 (100 PTS.) Grammar it.

Describe a context free grammar for the following languages. Clearly explain how they work and the role of each non-terminal. Unclear grammars will receive little to no credit.

11.A. (40 PTS.) $\left\{ a^i b^j c^k d^\ell e^t \;\middle|\; i, j, k, \ell, t \geq 0 \text{ and } i + j + k + t = \ell \right\}$.

11.B. (60 PTS.) (Harder.) $L = \{z \in \{a, b, c\}^* \mid \text{ there is a suffix } y \text{ of } z \text{ s.t. } \#_a(y) > \#_b(y)\}$.
(Hint: First solve for the case that z has no cs.)

12 (100 PTS.) As easy as 1,2,3,6.

Let $L = \left\{ a^i b^j c^k \;\middle|\; k = i + j \right\}$.

12.A. (20 PTS.) Prove that L is context free by describing a grammar for L.

12.B. (80 PTS.) Prove that your grammar is correct. (See extra problems for an example of how this is done.)

[1] Feel free to use IDK.