7 (100 PTS.) Draw me a giraffe.

For each of the following languages in **7.A.–7.C.**, draw an NFA that accepts them. Your automata should have a small number of states. Provide a short explanation of your solution, if needed.

7.A. (25 PTS.) All strings in $\{0, 1, 2\}^*$ such that at least one of the symbols 0, 1, or 2 occurs at most 4 times. (Example: 1200201220210 is in the language, since 1 occurs 3 times.)

7.B. (25 PTS.) $\left((01)^*(10)^* + 00\right)^* \cdot (1 + 00 + \varepsilon) \cdot (11)^*$.

7.C. (25 PTS.) All strings in $\{0, 1\}^*$ such that the last symbol is the same as the third last symbol. (Example: 1100101 is in the language, since the last and the third last symbol are 1.)

7.D. (25 PTS.) Use the power-set construction (also called subset construction) to convert your NFA from **7.C.** to a DFA. You may omit unreachable states.

8 (100 PTS.) Fun with parity.

Given $L \subseteq \{0, 1\}^*$, define $\mathit{even}_0(L)$ to be the set of all strings in $\{0, 1\}^*$ that can be obtained by taking a string in L and inserting an even number of 0's (anywhere in the string). Similarly, define $\mathit{odd}_0(L)$ to be the set of all strings x in $\{0, 1\}^*$ that can be obtained by taking a string in L and inserting an odd number of 0's.

(Example: if $01101 \in L$, then $01010000100 \in \mathit{even}_0(L)$.)

(Another example: if L is 1^*, then $\mathit{even}_0(L)$ can be described by the regular expression $(1^*01^*0)^*1^*$.)

The purpose of this question is to show that if $L \subseteq \{0, 1\}^*$ is regular, then $\mathit{even}_0(L)$ and $\mathit{odd}_0(L)$ are regular.

8.A. (30 PTS.) For each of the base cases of regular expressions $\emptyset$, ε, 0, and 1, give regular expressions for $\mathit{even}_0(L(r))$ and $\mathit{odd}_0(L(r))$.

8.B. (60 PTS.) Given regular expressions for $e_j = \mathit{even}_0(L(r_j))$ and $o_j = \mathit{odd}_0(L(r_j))$, for $j \in \{1, 2\}$, give regular expressions for

(i) $\mathit{even}_0(L(r_1 + r_2))$
(ii) $\mathit{odd}_0(L(r_1 + r_2))$
(iii) $\mathit{even}_0(L(r_1 r_2))$
(iv) $\mathit{odd}_0(L(r_1 r_2))$
(v) $\mathit{even}_0(L(r_1^*))$
(vi) $\mathit{odd}_0(L(r_1^*))$

Give brief justification of correctness for each of the above.

8.C. (10 PTS.) Using the above, describe (shortly) a recursive algorithm that given a regular expression r, outputs a regular expression for $\mathit{even}_0(L(r))$ (similarly describe the algorithm for computing $\mathit{odd}_0(L(r))$).

9 (100 PTS.) "+1".

Let binary(i) denote the binary representation of a positive integer i. (Note that the string binary(i) must start with a 1.)

Given a language $L \subseteq \{0, 1\}^*$, define $\mathrm{INC}(L) = \{\mathrm{binary}(i+1) \mid \mathrm{binary}(i) \in L\}$. For the time being assume that L does not contain any string of 1^*.

(Example: for $L = \{100, 101011, 1101\}$, we have $\mathrm{INC}(L) = \{101, 101100, 1110\}$.)

9.A. (30 PTS.) Given a DFA $M = (Q, \Sigma, \delta, s, A)$ for L, describe **informally** (in a few sentences) how to construct an NFA M_w for $\mathrm{INC}(L)$.

9.B. (30 PTS.) Given a DFA $M = (Q, \Sigma, \delta, s, A)$ for L, describe **formally** how to construct an NFA M' for $\mathrm{INC}(L)$.

9.C. (30 PTS.) Prove formally the correctness of your construction from (**9.B.**). That is, prove that $\mathrm{INC}(L) = L(M')$.

9.D. (10 PTS.) Describe formally how to modify the construction of M' from above, to handle that general case (without the above assumption) that L might also contain strings of the form 1^*. You do not need to provide a proof of correctness of the new automata.