HW 2

Due on **Wednesday, January 30, 2019 at 10am**

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.02**

Submission instructions as in previous homeworks.

4 (100 PTS.) Regularize this.

For each of the following languages over the alphabet $\{0, 1\}$, give a regular expression that describes that language, and briefly argue why your expression is correct.

4.A. (20 PTS.) All strings that contain the subsequence 101.

4.B. (20 PTS.) All strings that do not contain the subsequence 111.

4.C. (20 PTS.) All strings that start in 11 and contain 110 as a substring.

4.D. (20 PTS.) All strings that do not contain the substring 100.

4.E. (20 PTS.) All strings in which every nonempty maximal substring of consecutive 0s is of length 1. For instance 1001 is not in the language while 10111 is.

5 (100 PTS.) Then, shalt thou find two runs of three.

Let L be the set of all strings in $\{0, 1\}^*$ that contain the substrings 000 and 111.

5.A. (60 PTS.) Describe a DFA that over the alphabet $\Sigma = \{0, 1\}$ that accepts the language L. Argue that your machine accepts every string in L and nothing else, by explaining what each state in your DFA *means*.

You may either draw the DFA or describe it formally, but the states Q, the start state s, the accepting states A, and the transition function δ must be clearly specified.

5.B. (40 PTS.) Give a regular expression for L, and briefly argue why the expression is correct.

6 (100 PTS.) Construct This

Let L_1 and L_2 be regular languages over Σ accepted by DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$, respectively.

6.A. (30 PTS.)

Describe a DFA $M = (Q, \Sigma, \delta, s, A)$ in terms of M_1 and M_2 that accepts $L = L_1 \cup \overline{L_2} \cup \{\epsilon\}$. Formally specify the components Q, δ, s, and A for M in terms of the components of M_1 and M_2.

6.B. (30 PTS.)

Let $H_1 \subseteq Q_1$ be the set of states q such that there exists a string $w \in \Sigma^*$ where $\delta_1^*(q, w) \in A_1$. Consider the DFA $M' = (Q_1, \Sigma, \delta_1, s_1, H_1)$. What is the language $L(M')$? Formally prove your answer!

6.C. (40 PTS.) Suppose that for every $q \in A_2$ and $a \in \Sigma$, we have $\delta_2(q, a) = q$. Prove that $\epsilon \in L_2$ if and only if $L_2 = \Sigma^*$.