Problem 1. [Category: Design+Proof] Prove that the recursive languages are closed under the following operations:

- union
- intersection
- complement
- concatenation

Problem 2. [Category: Design+Proof] Prove that if \( L_1 \) and \( L_2 \) are recursive, then so is \( \text{SHUFFLE}(L_1, L_2) = \{w \mid w = \alpha_1 \beta_1 \alpha_2 \beta_2 \cdots \alpha_k \beta_k \text{ for some } k \geq 0 \text{ and strings } \alpha_1, \ldots, \alpha_k \text{ and } \beta_1, \ldots, \beta_k, \text{ such that } \alpha_1 \alpha_2 \cdots \alpha_k \in L_1 \text{ and } \beta_1 \beta_2 \cdots \beta_k \in L_2 \} \).

Problem 3. [Category: Design+Proof] Show that if \( L_1 \) and \( L_2 \) are recursively enumerable, then so is \( \text{SHUFFLE}(L_1, L_2) \).