**Problem 1.** [Category: Comprehension] Consider the following Turing machine $M$ over the input alphabet $\Sigma = \{a\}$. The reject state $q_{\text{rej}}$ is not shown, and all “missing” transitions are assumed to go to $q_{\text{rej}}$ as per our convention.

1. Describe the computation of $M$ on input $aaaa$ (4 a’s) as a sequence of configurations.

2. What language does $M$ recognize? Outline an informal justification for your answer. *Hint:* Recall that, $1 + 3 + 5 + \cdots + (2k - 1) = k^2$.

**Problem 2.** [Category: Design] Design a TM to recognize the language $\{a^i b^j c^k \mid k = i \ast j\}$.

**Problem 3.** [Category: Design] **Shifting symbols:** Design a TM $M_k$ such that given input $w \in \Sigma^*$, $M$ halts with the tape containing $\square^k w$. 

---

---