Problem 1. [Category: Comprehension] Describe the language defined by the following regular expressions.

1. $0^*1000(0+1)^*$
2. $1^*0^*+(00^*11^*00^*)^*+(11^*00^*11^*)^*$

Problem 2. [Category: Design] Give regular expressions over alphabet \{0, 1\} for each of the following languages. We won’t get to all of them in the lab.

1. All strings containing at least three 0s.
2. All strings containing at least two 0s and at least one 1.
3. All strings containing the substring 000.
4. All strings not containing the substring 000.
5. All strings in which every run on 0s has length at least 3.
6. Every string except 000. Hint: Don’t try to be clever.
7. All strings \(w\) such that in every prefix of \(w\), the number of 0s and 1s differ by at most 1.

Problem 3. [Category: Comprehension] To think at home: Given two regular expressions \(r\) and \(s\), we write \(r = s\) if \(L(r) = L(s)\). Which of the following is true?

1. \((0+1)^* = 0^* + 1^*\)
2. \((01 + 0)^*0 = 0(10 + 0)^*\)
3. \(1(01 + 1)^*0 = 11^*0(11^*0)^* = (1^+0)^+\)

Problem 4. [Category: Design] Design DFAs for the following languages over the alphabet \{0, 1\}.

1. \(L_1\) is the set of all strings that have 1 in every odd position.
2. \(L_2 = \{0w \mid |w| \text{ is odd} \} \cup \{1w \mid |w| \text{ is even} \}\)
3. \(L_3 = \{w \mid \text{every prefix } x \text{ of } w \text{ has } |\#_0(x) - \#_1(x)| \leq 2 \}\). Here, \#_0(y) and \#_1(y) are the number of 0’s and 1’s respectively in the string \(y\).

Problem 5. [Category: Design] Design a DFA that accepts all strings over the alphabet \{$, c, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, .$\} that correspond to valid currency amounts. A valid string is either
• a dollar sign followed by a number which has no leading 0's (unless the number is a single 0 by itself), optionally followed by a decimal point and exactly two decimal digits, OR
• a one or two-digit number with no leading 0's (unless the number is a single 0 by itself) followed by the cent sign c.

Thus, $432.63, $0, $0.02, $0.00, 47c, 2c, 0c are all accepted, but $021, $4.3, $8.63c, $0.0, $.02, 02c, 00c are not accepted.

Problem 6. [Category: Design] To think at home: Design DFAs for the following languages.

1. $(0 + 1)^*$
2. $\emptyset$
3. $\{\epsilon\}$
4. Every string except 000.
5. All strings containing the substring 000.
6. All strings not containing the substring 000.
7. All strings in which the reverse of the string is the binary representation of an integer divisible by 3.