Every RSO has officers — pres, treasurer,...

President's Club
↓
Non-President's Club can't be an RSO!

Barber [Russell]

Haircut Day

Licensed barber — cuts anyone's hair

DON'T EXIST

Cantor's Theorem: Any set \( X \)

Any function \( F: X \rightarrow 2^X \)

\( F \) is not surjective

\( \exists S \in 2^X \) s.t. \( F(x) \neq S \) for all \( x \in X \)

Proof: Call \( x \in X \) happy if \( x \in F(x) \)

sad \( x \notin F(x) \)

Let \( S = \) all sad elements of \( X = \{ x | x \notin F(x) \} \)

Suppose for argument \( F \) is surjection

Then \( \exists y \in X \) s.t. \( F(y) = S \)

For all \( x \): \( x \in S \iff x \in F(y) \iff x \notin F(x) \)

So \( y \notin F(y) \iff y \notin F(x) \)
SelfReject = ∃<M> | M rejects <M>

Suppose there is a program ST2 s.t.
Given another source <M>
If M rejects <M> ST2 returns YES
If M accepts <M> or hangs on <M>
ST2 returns NO

ST2 accepts <M> ⇔ M rejects <M>
Set M = ST2
ST2 accepts <ST2> ⇔ ST2 rejects <ST2>
Contradiction!

SelfAccept = ∃<M> | M accepts <M>

Suppose there is a machine SA that decides SelfAccept
Build new machine SA':

SA'(w):
ok ← SA(w)
return ~ok

SA accepts <M> ⇔ M does not accept <M>
SA' accepts <SA'> ⇔ SA' does not accept <SA'>
Halting Problem: Given \( \langle M \rangle \) and \( w \)

Does \( M \) halt if \( w \) is the input to \( M \)?

\[ \text{Collatz}(n): \]
- if \( n = 1 \) halt
- else if \( n \) even \( \text{Collatz}(n/2) \)
- else \( \text{Collatz}(3n+1) \)

Suppose \( H \) decides \( \text{HALT} = \{ \langle M \rangle \cdot w \mid M \text{ halts on } w \} \)

\[ \text{SH}(x): \]

- first check \( x = \langle M \rangle \) for some \( M \)
- else reject
- if \( H(x, x) \)
  - return \( \text{true} \)
- else return \( \text{false} \)

\( \text{Self Halts} = \{ \langle M \rangle \mid M \text{ halts on } \langle M \rangle \} \)

Suppose there is a machine \( SH \)

that decides \( \text{Self Halts} \)

Build new machine \( SA' \):

\[ \text{SH}'(w): \]

- run \( \text{SH}(w) \)
- return \( \text{false} \)

\( \text{SH}' \) accepts \( \langle M \rangle \) \iff \( M \) does not accept \( \langle M \rangle \)

\( \text{SH}' \) accepts \( \langle SA' \rangle \) \iff \( SA' \) does not accept \( \langle SA' \rangle \)