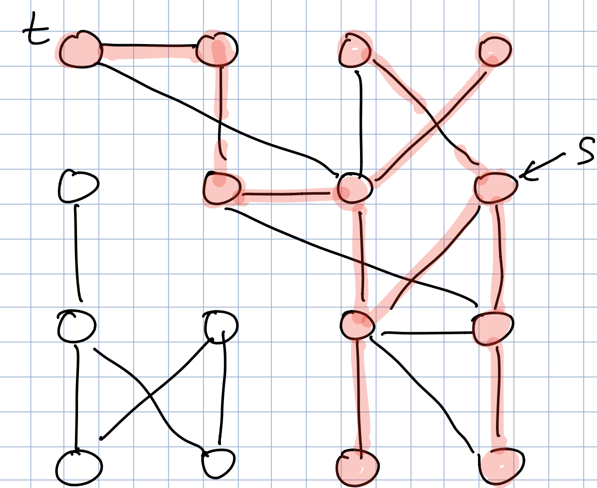
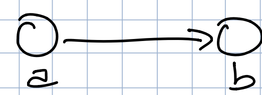
Whatever-first search

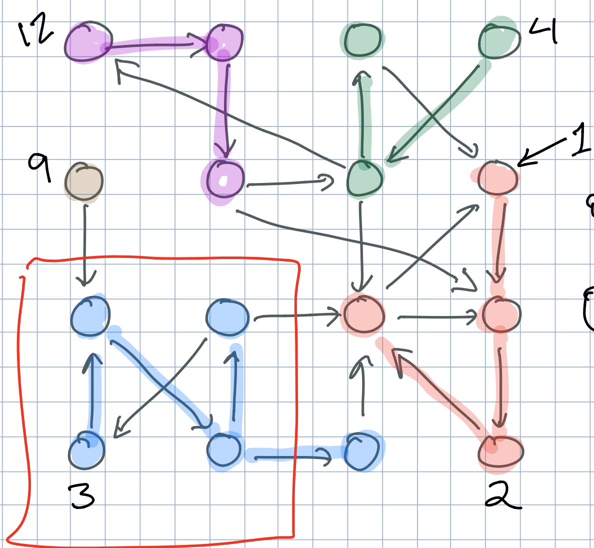


Depth-first search

Directed graphs

reachability isn't symmetric





over estimate

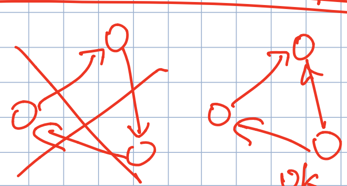
$O(E)$

```
DFS(v):
  mark v
  PREVISIT(v)
  for each edge v→w
    if w is unmarked
      parent(w) ← v
      DFS(w)
  POSTVISIT(v)
```

```
DFSALL(G):
  PREPROCESSING(G)
  for all vertices v
    unmark v
  for all vertices v
    if v unmarked
      DFS(v)
```

Directed Acyclic Graph

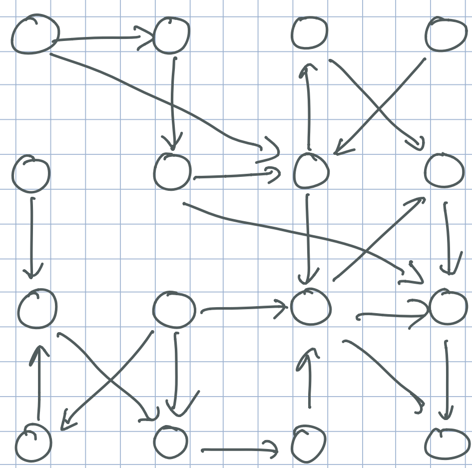
$O(V+E)$ time



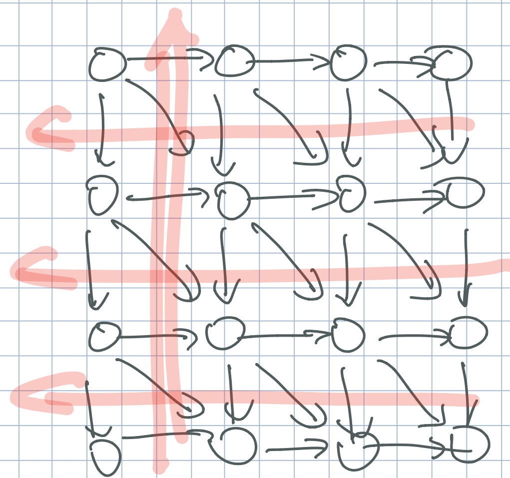
Strongly Connected

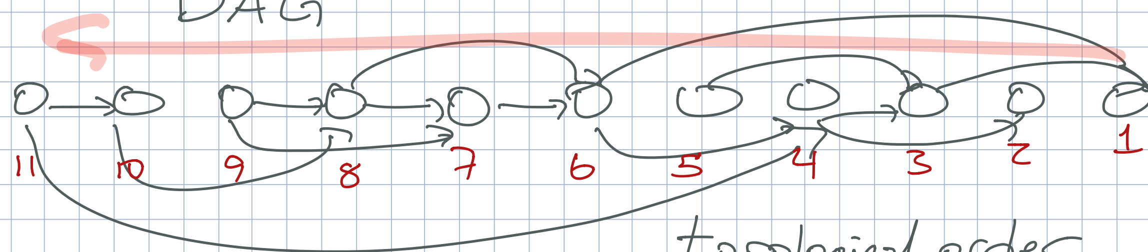
$u \rightsquigarrow v$ and $v \rightsquigarrow u$

for all $u \neq v$



DAG





topological order sort

```
DFS(v):
  mark v
  PREVISIT(v)
  for each edge v→w
     if w is unmarked
        parent(w) ← v
        DFS(w)
  POSTVISIT(v)
```

```
MEMOIZE(x):
  if value[x] is undefined
     init value[x]
     for all subprobs y of x
        MEMOIZE(y)
        update value[x] ← value[y]
     finalize value[x]
  return value[x]
```

Memoized recursion IS DFS in DAG

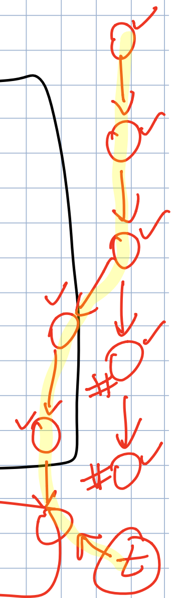
Dynamic programming IS top sort + iterative traversal

DFS(v):

mark v
for each edge $v \to w$
  if w is unmarked
    parent(w) ← v
    DFS(w)
v.done ← clock
clock ← clock + 1

DFSALL(G):

clock ← 0
for all vertices v
  unmark v
Add vertex s
for all $v \in V$
  add $s \to v$
DFS(s)



Lemma: After DFSALL [iff G is a dag]

for all $v \to w$

v.done > w.done

Proof: Let t be the vertex s.t. t.done = 0

Claim: t has no out edges. t is a sink

Spose $t \to z$

DFS(t) calls DFS(z) unless z marked

- z is not already marked
  call DFS(z) ⟶ z.done ← ?
  t.done ← ? + 1 > z.done ≥ 0
  t.done > 0 ✗
- z is already marked and z.done undef.
  z can reach t   t can reach z
  $t \to z$
  cycle! ✗

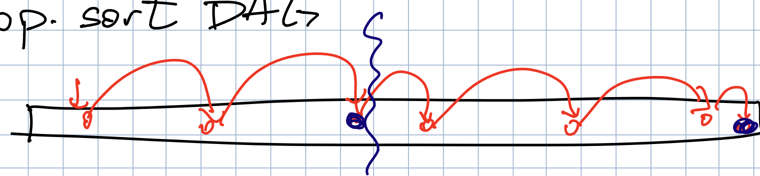
Longest Path in a DAG

Input: DAG $G=(V,E)$ $\quad \ell : E \to \mathbb{R}$ edge length

Want: total length of longest path in G

$$v_0 \to v_1 \to \cdots \to v_k$$

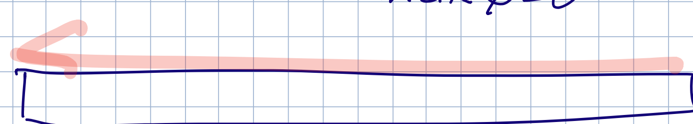
Algo: (1) Top. sort DAG



$LLP(i)$ = length of longest path in G starting at vertex i

$$LLP(i) = \begin{cases} \cancel{0 \qquad i = v} \\ \max\{\ell(i \to j) + LLP(j) \mid i \to j\} & \text{o/w} \end{cases}$$

$\max \emptyset = 0$

LLP 

$O(V+E)$