Mille viae ducunt homines per saecula Romam.
[A thousand roads lead men forever to Rome.]
— Alain de Lille, *Liber Parabolarum* (1175)

I study my Bible as I gather apples.
First I shake the whole tree, that the ripest might fall.
Then I climb the tree and shake each limb,
and then each branch and then each twig,
and then I look under each leaf.
— attributed to Martin Luther (c. 1500)

Thus you see, most noble Sir, how this type of solution bears little relationship to mathematics, and I do not understand why you expect a mathematician to produce it, rather than anyone else, for the solution is based on reason alone, and its discovery does not depend on any mathematical principle. Because of this, I do not know why even questions which bear so little relationship to mathematics are solved more quickly by mathematicians than by others.
— Leonhard Euler, describing the Königsburg bridge problem in a letter to Carl Leonhard Gottlieb Ehler (April 3, 1736)
Figure 5.5. Two drawings of the same disconnected planar graph with 13 vertices, 19 edges, and two components.

\[
F_n = \begin{cases} 
0 & \text{if } n = 0, \\
1 & \text{if } n = 1, \\
F_{n-1} + F_{n-2} & \text{otherwise}, 
\end{cases}
\]

\[
Edit(i, j) = \begin{cases} 
i & \text{if } j = 0 \\
j & \text{if } i = 0 \\
\min \left\{ \begin{array}{l}
\text{Edit}(i - 1, j) + 1, \\
\text{Edit}(i, j - 1) + 1, \\
\text{Edit}(i - 1, j - 1) + [A[i] \neq B[j]]
\end{array} \right\} & \text{otherwise}
\end{cases}
\]
Figure 5.10. An adjacency list for our example graph.

Figure 5.12. An adjacency matrix for our example graph.
<table>
<thead>
<tr>
<th>Space</th>
<th>Standard adjacency list (linked lists)</th>
<th>Fast adjacency list (hash tables)</th>
<th>Adjacency matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Theta(V + E)$</td>
<td>$\Theta(V + E)$</td>
<td>$\Theta(V^2)$</td>
</tr>
<tr>
<td>Test if $uv \in E$</td>
<td>$O(1 + \min{\deg(u), \deg(v)}) = O(V)$</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Test if $u \rightarrow v \in E$</td>
<td>$O(1 + \deg(u)) = O(V)$</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>List $v$’s (out-)neighbors</td>
<td>$\Theta(1 + \deg(v)) = O(V)$</td>
<td>$\Theta(1 + \deg(v)) = O(V)$</td>
<td>$\Theta(V)$</td>
</tr>
<tr>
<td>List all edges</td>
<td>$\Theta(V + E)$</td>
<td>$\Theta(V + E)$</td>
<td>$\Theta(V^2)$</td>
</tr>
<tr>
<td>Insert edge $uv$</td>
<td>$O(1)$</td>
<td>$O(1)^*$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Delete edge $uv$</td>
<td>$O(\deg(u) + \deg(v)) = O(V)$</td>
<td>$O(1)^*$</td>
<td>$O(1)$</td>
</tr>
</tbody>
</table>

**WhateverFirstSearch(s):**
- put $s$ into the bag
- while the bag is not empty
  - take $v$ from the bag
  - if $v$ is unmarked
    - mark $v$
    - for each edge $vw$
      - put $w$ into the bag

**WhateverFirstSearch(s):**
- put $(s, s)$ in bag
- while the bag is not empty
  - take $(p, v)$ from the bag
  - if $v$ is unmarked
    - mark $v$
    - $\text{parent}(v) \leftarrow p$
  - for each edge $vw$
    - put $(v, w)$ into the bag

Bag: stores a set of vertices
- insert
- take
- out

- **(*)**
- **(†)**
- **(***)**
**WFSAll(G):**
for all vertices \( v \)
  unmark \( v \)
for all vertices \( v \)
  if \( v \) is unmarked
    WhateverFirstSearch(\( v \))

**COUNTAndLABEL(G):**
\[
count \leftarrow 0
\]
for all vertices \( v \)
  unmark \( v \)
for all vertices \( v \)
  if \( v \) is unmarked
    \[
    count \leftarrow count + 1
    \]
    LABELOne(\( v, count \))
return count

**LABELOne(\( v, count \)):**
while the bag is not empty
  take \( v \) from the bag
  if \( v \) is unmarked
    mark \( v \)
    \[
    comp(v) \leftarrow count
    \]
    for each edge \( vw \)
      put \( w \) into the bag