$DFA = (Q, \Sigma, s, A, \delta)$

Q - states

Σ - alphabet

$s \in Q$ - start state

$A \subseteq Q$ - accepting states

$\delta : Q \times \Sigma \to Q$ - transition function

$\delta^* : Q \times \Sigma^* \to Q$

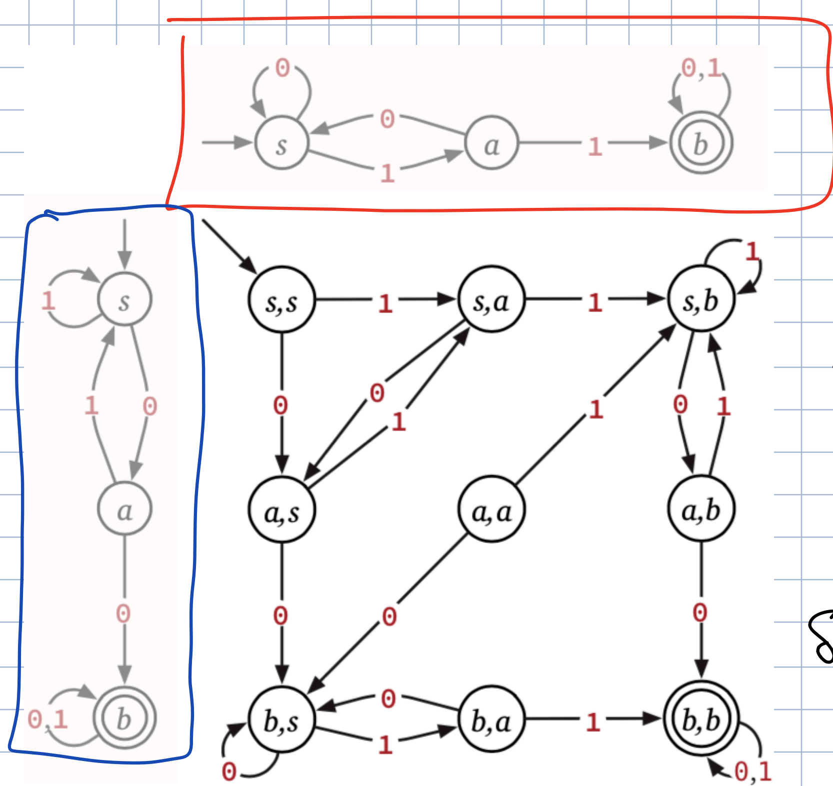
$$\delta^*(q, w) = \begin{cases} q & \text{if } w = \varepsilon \\ \delta^*(\delta(q,a), x) & \text{if } w = ax \end{cases}$$

M accepts w iff $\delta^*(s, w) \in A$

$\cap$

$\wedge$

Product construction



$Q = Q_1 \times Q_2$

$= \{(q_1, q_2) \mid q_1 \in Q_1 \text{ and } q_2 \in Q_2\}$

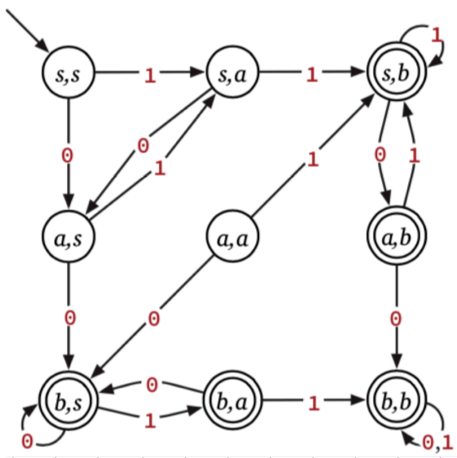
$s = (s_1, s_2)$

$A = \{(q_1, q_2) \mid q_1 \in A_1 \text{ and } q_2 \in A_2\}$

$= A_1 \times A_2$

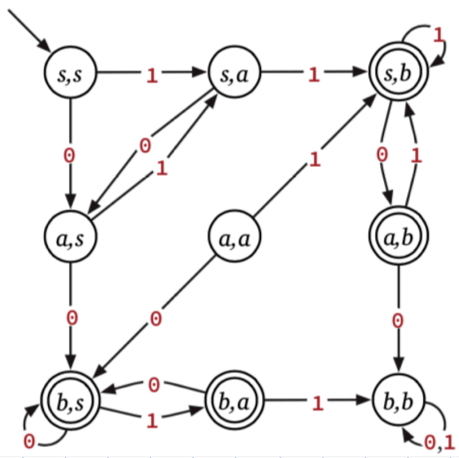
$\delta((q_1, q_2), a)$

$= (\delta_1(q_1, a), \delta_2(q_2, a))$

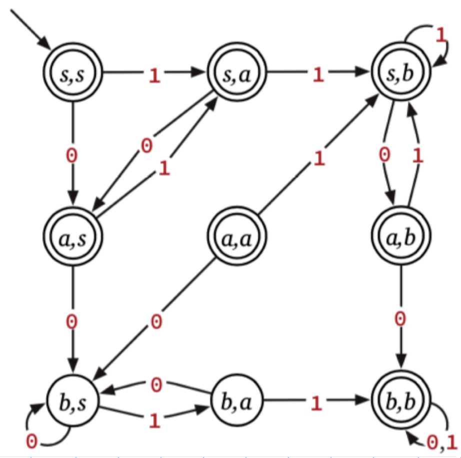


OR

UNION



XOR



$\Longrightarrow$

if contains 00 then contains 11

$Q = Q_1 \times Q_2$

$= \{(q_1, q_2) \mid q_1 \in Q_1 \text{ and } q_2 \in Q_2\}$

$s = (s_1, s_2)$

$A = \{(q_1, q_2) \mid q_1 \in A_1 \text{ and } q_2 \in A_2\}$

$= A_1 \times A_2$

$\delta((q_1, q_2), a)$

$= (\delta_1(q_1, a), \delta_2(q_2, a))$

Claim: $L(M) = L(M_1) \cap L(M_2)$

Key Lemma:

$\delta^*((p,q), w) = (\delta_1^*(p,w), \delta_2^*(q,w))$

Proof: Let p, q be arbitrary states

Let w be arb. string

Assume for all x shorter than w that and all states p', q'

$\delta^*((p',q'), x) = (\delta_1^*(p',x), \delta_2^*(q',x))$

Two cases:

- If $w = \varepsilon$

  $\delta^*((p,q), \varepsilon) = (p,q)$

  $= (\delta_1^*(p,\varepsilon), \delta_2^*(q,\varepsilon))$

- If $w = ax$

  $\delta^*((p,q), ax) =$

  $\delta^*(\delta((p,q),a), x) =$

  $\delta^*((\underbrace{\delta_1(p,a)}_{p'}, \underbrace{\delta_2(q,a)}_{q'}), x)$

  $(\delta_1^*(\delta_1(p,a), x), \delta_2^*(\delta_2(q,a), x))$

Thus, $\delta^*((p,q), w) = (\delta_1^*(p,w), \delta_2^*(q,w))$

If L, L' are recognized by DFA's $\Longleftrightarrow$ regular

Then $L \cap L'$

$L \cup L'$

$\boxed{L \setminus L'}$

$L \oplus L'$ are accepted by DFA's

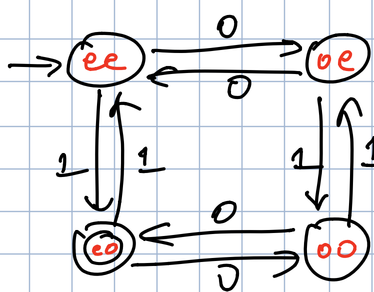
$\Sigma^* \setminus L = \overline{L}$

If L, L' are regular $\Longleftrightarrow$ automatic

Then $L \cup L'$

$L \cdot L'$

L^* are regular



$\delta^*(s, 0001) = oo$

$\delta^*(s, 1101) = oo$

$0001x \in L$

$\Longleftrightarrow$

$1101x \in L$

$01 \cdot 0 \in L_{eo}$

$0011 \cdot 0 \notin L_{eo}$

For any language L

For any strings x and y

If there is a DFA that accepts L s.t.

x and y lead to same state

Then

for all z

$xz \in L \Longleftrightarrow yz \in L$

If there is a string z s.t.

$xz \in L \Leftrightarrow yz \notin L$

Then for any DFA that accepts L

x and y lead to different states

01
101 —
010 —
1010

$L = \{ w \mid \#(10,w) \text{ even}, \#(1,w) \text{ odd} \}$

~~10~~1 01 $\in L$
010 01 $\notin L$

Fooling set: Any pair in set has a distinguishing suffix

If L has an infinite foolingset $\iff$ L is not regular

$\{0^n 1^n \mid n \geq 0\}$