Why are we here?

Theoretical computer science

What can be computed?
... quickly? How?
... under resource constraints?
or can't?

Computers are stupid.
People are clever. Clever is bad.

Today: Strings

Definition: A string is either
- nothing
- \((a,x)\) where \(a \in \Sigma\) “symbol”
  where \(x\) is a string.

“empty string” \(\varepsilon\)
\(a \cdot x \quad ax\)

STRING

\((\Sigma, \varepsilon, \cdot, (\cdot)^{\cdot}, \varepsilon, (\cdot)^{\cdot})\)

set of all strings over \(\varepsilon\) \(\rightarrow \varepsilon^*\)
Length - “# of symbols”

\[ |w| = \begin{cases} 
0 & \text{if } w = \varepsilon \\
1 + |x| & \text{if } w = ax 
\end{cases} \]

Concatenation - “write one after the other”

\( (\text{FOOT}) \bullet \text{BALL} = (\text{FOOTBALL}) \)

\[ w \cdot x = \begin{cases} 
x & \text{if } w = \varepsilon \\
\emptyset \cdot (y \cdot x) & \text{if } w = ay 
\end{cases} \]

\[ 3 \cdot 3 = 9 \]

\[ 3 \cdot \text{BALL} = \text{BALL} \quad \text{FOOT} \cdot 3 = \text{FOOT} \]

\[ |w \cdot x| = |w| + |x| \quad \text{for all strings } w \text{ and } x \]

**Proof:**

Let \( w \) and \( x \) be arbitrary strings.

Assume for all strings \( y \) shorter than \( w \) that \( |y \cdot x| \leq |y| + |x| \)

There are two cases:

- If \( w = \varepsilon \)

\[ |w \cdot x| = |\varepsilon \cdot x| \]
\[ = |x| \quad [\text{by definition of } \varepsilon] \]
\[ = 0 + |x| \quad [\text{duh}] \]
\[ = |\varepsilon| + |x| = |w| + |x| \quad [\text{by def. } |\varepsilon|] \]
If $w = ay$

\[ |w \cdot x| = |(ay) \cdot x| \]

\[ = |e \cdot (y \cdot x)| \quad \text{[by def. of $e$]} \]

\[ = 1 + |y \cdot x| \quad \text{[by def. of $1$]} \]

\[ = 1 + |y| + |x| \quad \text{[by IH]} \]

\[ = |ay| + |x| = |w| + |x| \quad \text{[by def. of $1$]} \]

Therefore $|w \cdot x| = |w| + |x|$