This homework is optional. However, similar undecidability questions may appear on the final exam, so we still strongly recommend treating at least those questions as regular homework. Solutions will be released next Tuesday as usual.

1. Let $M$ be a Turing machine, let $w$ be an arbitrary input string, and let $s$ be an integer. We say that $M$ accepts $w$ in space $s$ if, given $w$ as input, $M$ accesses only the first $s$ (or fewer) cells on its tape and eventually accepts.

   *(a) Sketch a Turing machine/algorithm that correctly decides the following language:

   $\text{SQUARESPACE} = \{ \langle M, w \rangle \mid M \text{ accepts } w \text{ in space } |w|^2 \}$

   (b) Prove that the following language is undecidable:

   $\text{SOME SQUARESPACE} = \{ \langle M \rangle \mid M \text{ accepts at least one string } w \text{ in space } |w|^2 \}$

2. Consider the following language:

   $\text{PICKY} = \{ \langle M \rangle \mid M \text{ accepts at least one input string and } M \text{ rejects at least one input string} \}$

   (a) Prove that PICKY is undecidable.

   (b) Sketch a Turing machine/algorithm that accepts PICKY.