Problem 1. [Category: Proof] Suppose we are given both an undirected graph $G$ with weighted edges and a minimum spanning tree $T$ of $G$. In all cases, the input to your algorithm is the edge $e$ and its new weight; your algorithms should modify $T$ so that it is still a minimum spanning tree. Of course, we could just recompute the minimum spanning tree from scratch in $O(|E| + |V| \log |V|)$ time, but you can do better.

1. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \in T$ is decreased.

2. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \not\in T$ is increased.

3. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \in T$ is increased.

4. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge $e \not\in T$ is decreased.

Problem 2. [Category: Design] Let $G = (V, E)$ be an undirected graph where each edge has a weight from the set $\{1, 10, 25\}$. Describe a linear-time algorithm to find an MST of $G$. 