Problem 1. [Category: Comprehension] Consider the following Turing machine $M$ over the input alphabet $\Sigma = \{a\}$. The reject state $q_{\text{rej}}$ is not shown, and all “missing” transitions are assumed to go to $q_{\text{rej}}$ as per our convention.

1. Describe the computation of $M$ on input $aaaa$ (4 a’s) as a sequence of configurations.

2. What language does $M$ recognize? Outline an informal justification for your answer. Hint: Recall that, $1 + 3 + 5 + \cdots + (2k - 1) = k^2$.

Problem 2. [Category: Design] Design a TM to recognize the language $\{a^i b^j c^k \mid k = i \ast j\}$.

Problem 3. [Category: Design] Shifting symbols: Design a TM $M_k$ such that given input $w \in \Sigma^*$, $M$ halts with the tape containing $\square^k w$. 

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