Recall the following notions: Sets, set building notation, subset, proper subset, empty set, Venn diagram, Cartesian product of sets, power set of a set.

**Problem 1.** [Category: Comprehension] For each of the following statements answer **True**, **False**, or **Meaningless**.

- \{a, b, c\} \cap \{d, e\} = \emptyset
- \{a, b, c\} \cap \{d, e\} = \{\emptyset\}
- \{a, b, c\} \cup \{d, a, e\} = \{a, b, c, d, a, e\}
- \emptyset \in \{\emptyset, a\}
- S \in \mathcal{P}(S)$, where $S$ is a set and $\mathcal{P}(S)$ is the powerset of $S$
- $a \in \mathcal{P}\{\{a\}\}$
- $\{a, b\} + \{c, d\} = \{a, b, c, d\}$
- $\{\{a, a\}\} = \{a, a\}$
- $\{\{a\}, \{a\}\} = \{a, a\}$
- $\{a, b\} \times \{c\} = \{(a, b), (b, b)\}$
- $\{a, b\} \times \{c, d\} = \{c, d\} \times \{a, b\}$

**Problem 2.** [Category: Comprehension+Proof] Let us define a set $U_n$ inductively as follows.

- $U_1 = \{1\}$
- $U_i = U_{i-1} \cup \{\max(U_{i-1}) + 2(i - 1) + 1\}$

Answer the following questions about the set $U_n$.

1. What is $U_2$? What is $U_3$?
2. Is $U_{n-1} \in U_n$?
3. What is $U_n$? Prove your answer.

**Problem 3.** [Category: Comprehension+Proof] For a string $w \in \{0, 1\}^*$, $w^c$ is inductively defined as follows.

$$w^c = \begin{cases} 
\epsilon & \text{if } w = \epsilon \\
1 & \text{if } w = 0 \\
0 & \text{if } w = 1 \\
(a^c)(u^c) & \text{if } w = au \text{ where } a \in \{0, 1\}, u \in \{0, 1\}^*
\end{cases}$$
1. What is $(10101)^c$?

2. Prove that for any strings $u, v \in \{0, 1\}^*$, $u^c \cdot v^c = (u \cdot v)^c$.

3. Recall $w^R$ denotes the reverse of string $w$ defined as

$$w^R = \begin{cases} 
\epsilon & \text{if } w = \epsilon \\
(u^R)^\cdot a & \text{if } w = a \cdot u \text{ where } a \in \Sigma, \ u \in \Sigma^*
\end{cases}$$

Prove that $(w^c)^R = (w^R)^c$. 