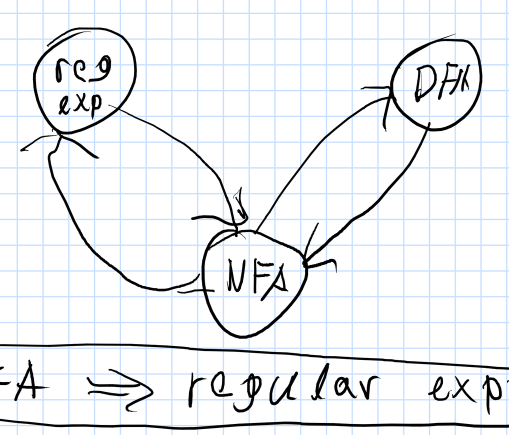
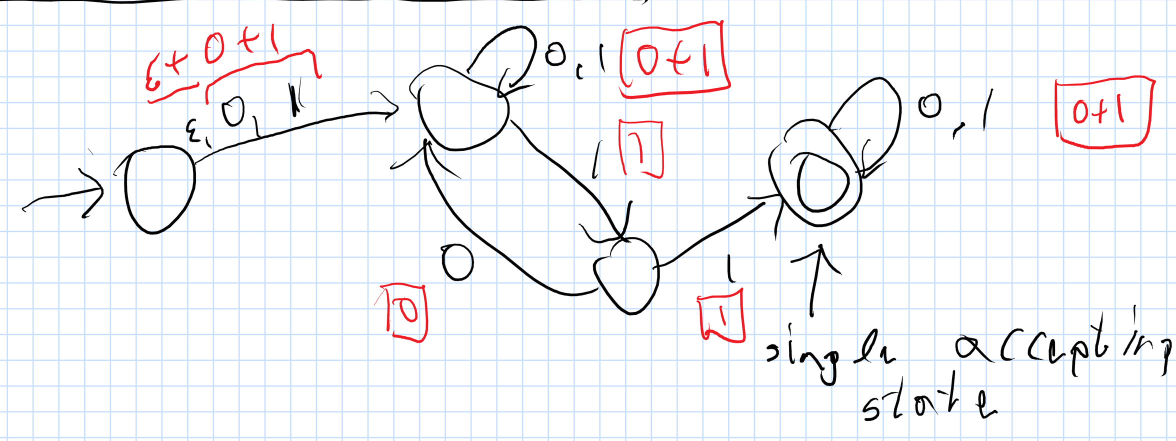
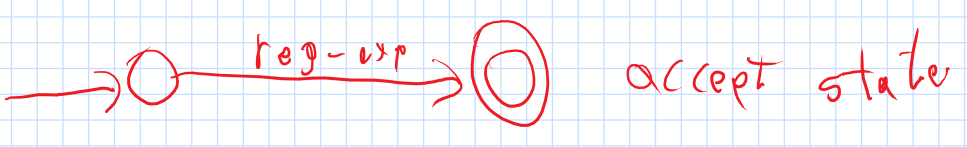
Lecture 6 9/8/22

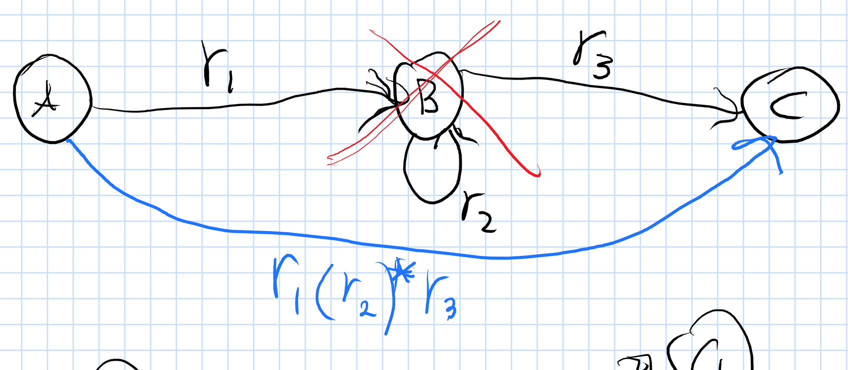


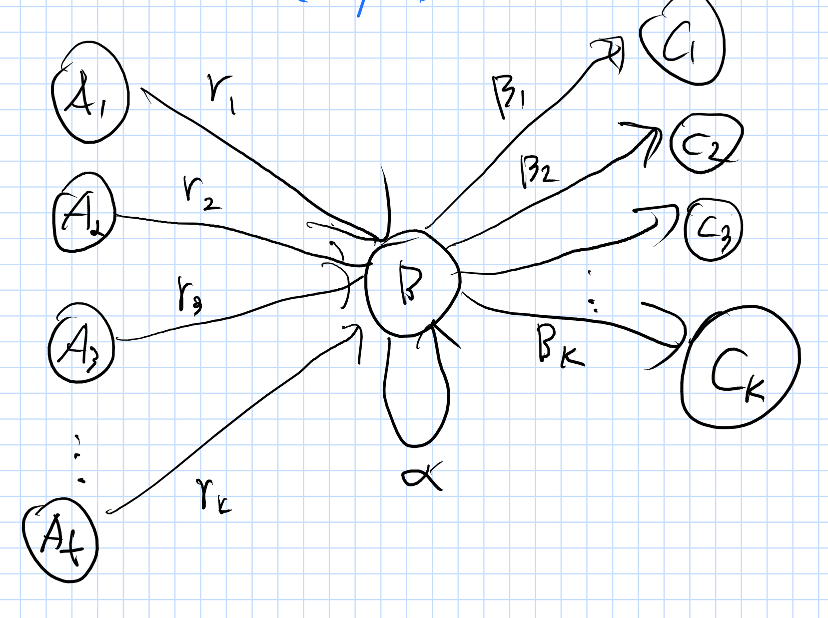
NFA $\Rightarrow$ regular expression



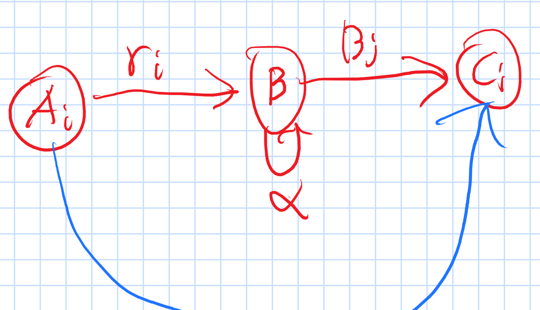
$\Downarrow$ convert edges labels to reg expressions



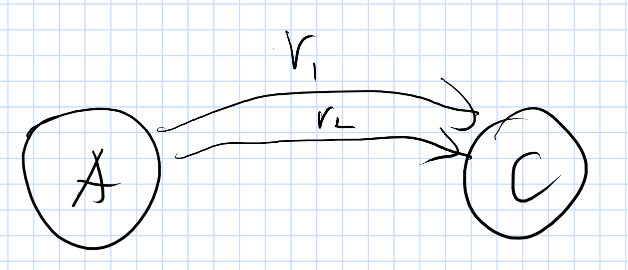


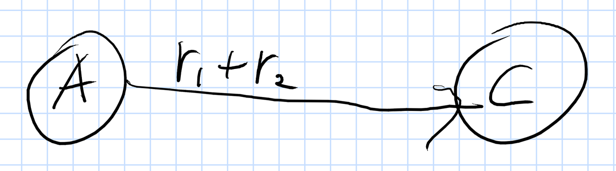


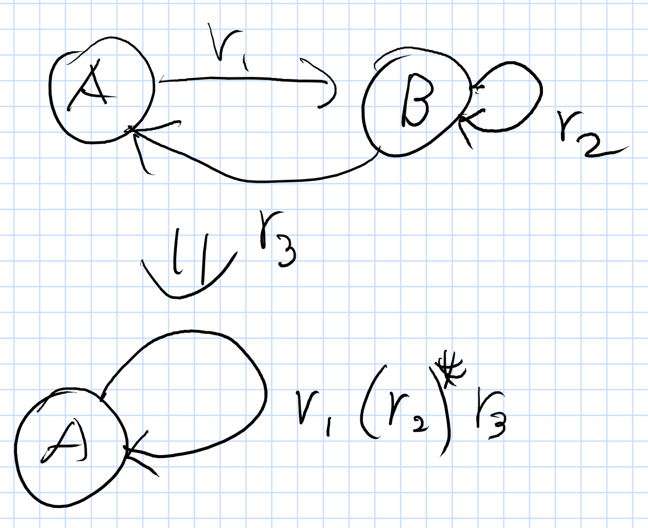
$\forall i, j$



$r_i \alpha^* B_j$







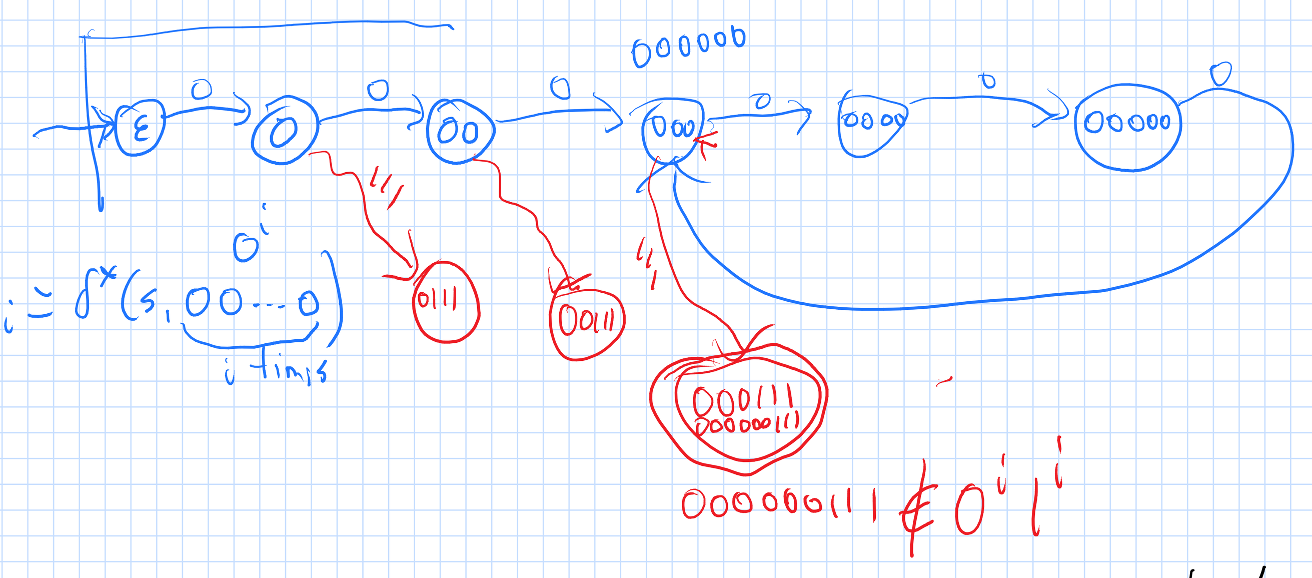
Non-regularity

$L = \{0^i 1^i \mid i \geq 0\} = \{\varepsilon, 01, 0011, 000111, \ldots\}$

L is regular $\Leftarrow$ Assumed for the sake of contradiction

$\Rightarrow \exists$ DFA $M = (Q, \Sigma, \delta, s, A)$

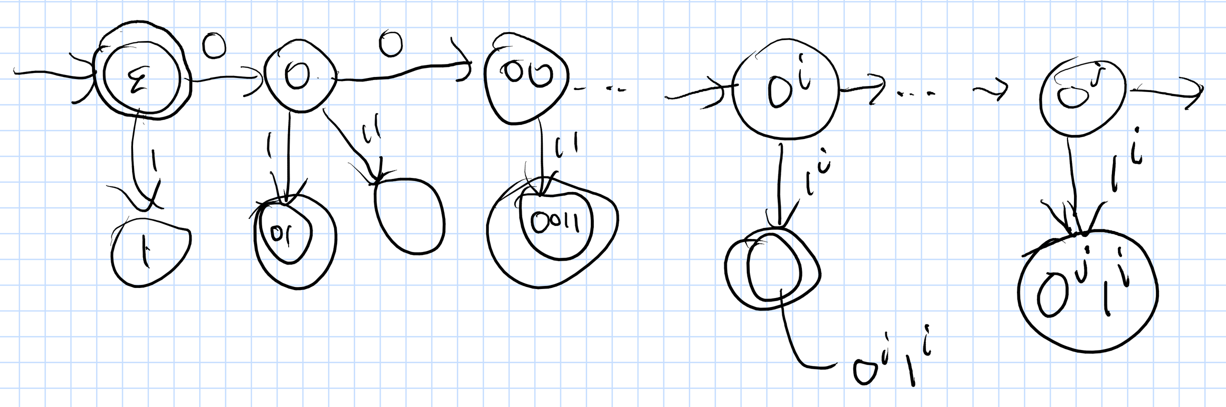
$L = L(M)$



$q_i := \delta^*(s, \underbrace{00\cdots0}_{i \text{ times}})$ 0^i

$000000111 \notin 0^i 1^i$

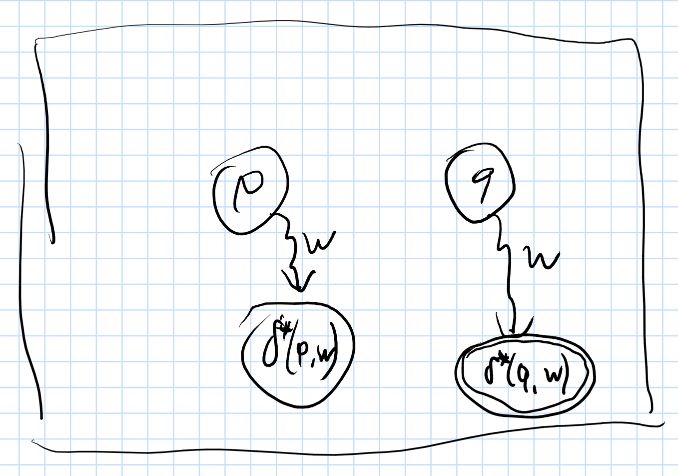
$i \neq j$



$\Rightarrow$ D has infinite # of states

$\Rightarrow$ Contradiction to the finiteness of D. ∎

Lemma The language $0^i 1^i = \{0^i 1^i \mid i \geq 0\}$ is not regular.



Def A string w distinguish states p and q in a DFA

$M = (Q, \Sigma, \delta, s, A)$

if $\delta^*(p,w) \in A$ and $\delta^*(q,w) \notin A$

or $\delta^*(p,w) \notin A$ and $\delta^*(q,w) \in A$.

The language L distinguish between strings $x, y \in \Sigma^*$ using a string w if

$\delta^*(s,x) = p$ $\quad$ $\delta^*(s,y) = q$

$\delta^*(p,w) \in A$ and $\delta^*(q,w) \notin A$

$\delta^*(\delta^*(s,x), w) = \delta^*(s, xw) \Rightarrow xw \in L$

$\delta^*(\delta^*(s,y), w) = \delta^*(s, yw) \notin A \Rightarrow yw \notin L$

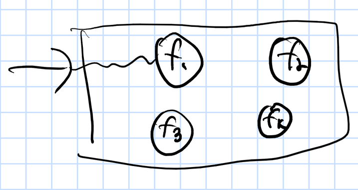
F: set of strings s.t. any pair of them are distinguishable by L.

$F = \{f_1, f_2, f_3 \ldots f_k\}$

$\forall i, j$ $\exists w_{ij}$ $f_i w_{ij} \in L$ and $f_j w_{ij} \notin L$

or $f_i w_{ij} \notin L$ and $f_j w_{ij} \in L$.

M a DFA for L

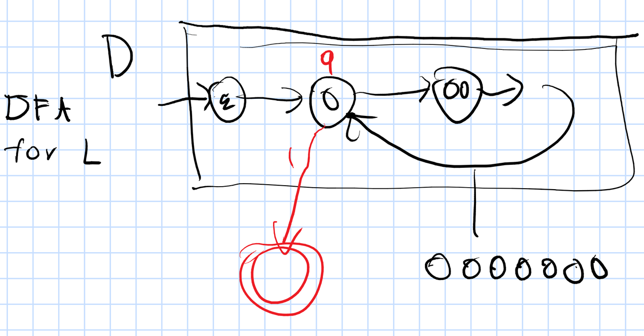


Any DFA for L must have at least $|F|$ different states.

Def For a language L, a set F of strings, s.t. any pair of strings in F is distinguishable, is a fooling set.

Labeling/naming set.

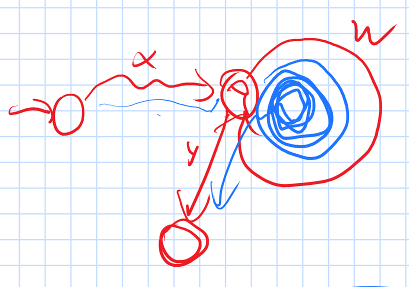
$L = \{0^i 1^i \mid i \geq 0\}$



$\delta^*(s,x) = q$

$\delta^*(s,xw) = q$

x



$\delta^*(s,xy) \in A)$

$\delta^*(s,xw^i y) \in A$

Pumping Lemma

$L = \{w \in \{0,1\}^* \mid \#_0(w) = \#_1(w)\}$

Lemma

L is not regular.

Proof Assume L is regular

$L \cap 0^* 1^* = \{0^i 1^i \mid i \geq 0\} \Leftarrow$ NOT reg!

must be regular because reg lang are closed under intersections.

Contradiction!