24.4.2
The consistency of execution

The variables of φ

Variables:

$\left\langle q_j, b_j, q'_j, b'_j, d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_1: The input is encoded correctly

φ_1 asserts (is true iff) the variables are set T/F indicating that **M** starts in state q_0 at time **0** with tape contents containing **x** followed by blanks. Let $x = x_1 x_2 \ldots x_n$

$$
\begin{aligned}
\varphi_1 = \quad & S(q_0, 0) && \text{// state at time } 0 \text{ is } q_0 \\
& \bigwedge_{h=1}^{n} T(x_h, h, 0) && \text{// at time } 0 \text{ cells } 1 \text{ to } n \text{ have value } x_1 \text{ to } x_n \\
& \wedge \bigwedge_{h=n+1}^{p(n)} T(\sqcup, h, 0) && \text{// all remaining cells are blank} \\
& \wedge H(1, 0) && \text{// The head is at time } 0 \text{ at start of tape}
\end{aligned}
$$

φ_2: M is in exactly one state at any point in time

φ_2 asserts M in exactly one state at any time i:

$$\varphi_2 = \bigwedge_{i=0}^{p(|x|)} \Big(\oplus \big(S(q_0, i), S(q_1, i), \ldots, S(q_{|Q|}, i) \big) \Big)$$

Variables:

$\left\langle q_j, b_j, q_j', b_j', d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_3: Each tape cell holds a unique symbol at any time

φ_3 asserts that each tape cell holds a unique symbol at any given time.

$$\varphi_3 = \bigwedge_{i=0}^{p(|x|)} \bigwedge_{h=1}^{p(|x|)} \oplus(T(b_1, h, i), T(b_2, h, i), \ldots, T(b_{|\Gamma|}, h, i))$$

For each time i and for each cell position h exactly one symbol $b \in \Gamma$ at cell position h at time i

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Variables:

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$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_4: tape head of M is in exactly one position at any time i

φ_4 asserts that the read/write head of M is in exactly one position at any time i

$$\varphi_4 = \bigwedge_{i=0}^{p(|x|)} (\oplus (H(1, i), H(2, i), \ldots, H(p(|x|), i)))$$

Variables:

$\left\langle q_j, b_j, q'_j, b'_j, d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_5: M accepts the input

φ_5 asserts that M accepts

- ▶ Let q_a be unique accept state of M
- ▶ without loss of generality assume M runs all $p(|x|)$ steps

$$\varphi_5 = S(q_a, p(|x|))$$

State at time $p(|x|)$ is q_a the accept state.

If we don't want to make assumption of running for all steps

$$\varphi_5 = \bigvee_{i=1}^{p(|x|)} S(q_a, i)$$

which means M enters accepts state at some time.

φ_6: M executes a unique instruction at each time

φ_6 asserts that M executes a unique instruction at each time

$$\varphi_6 = \bigwedge_{i=0}^{p(|x|)} \oplus(I(1,i), I(2,i), \ldots, I(m,i))$$

where m is max instruction number.

Variables:

$\left\langle q_j, b_j, q_j', b_j', d_j \right\rangle$: jth instruction of M

$I(j,i)$: Instruction j was issued at time i.

$H(h,i)$: The head is at location h at time i.

$T(c,h,i)$: The tape at location h at time i stored the character c.

φ_7: Tape changes only because of the head writing something

φ_7 ensures that variables don't allow tape to change from one moment to next if the read/write head was not there.

"If head is **not** at position h at time i then at time $i + 1$ the symbol at cell h must be unchanged"

$$\varphi_7 = \bigwedge_i \bigwedge_h \bigwedge_{b \neq c} \left(\overline{H(h, i)} \Rightarrow \overline{T(b, h, i) \bigwedge T(c, h, i+1)} \right)$$

since $A \Rightarrow B$ is same as $\neg A \vee B$, rewrite above in CNF form

$$\varphi_7 = \bigwedge_i \bigwedge_h \bigwedge_{b \neq c} \left(H(h, i) \vee \neg T(b, h, i) \vee \neg T(c, h, i+1) \right)$$

φ_8: Transitions are done from correct states

jth instruction of M: $< q_j, b_j, q'_j, b'_j, d_j >$

$$\varphi_8 = \bigwedge_i \bigwedge_j (I(j, i) \Rightarrow S(q_j, i))$$

If instruction j is executed at time i then state at time i must be q_j.

Variables:

$\left\langle q_j, b_j, q'_j, b'_j, d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_9: Transitions are done into correct state

jth instruction of M: $< q_j, b_j, q_j', b_j', d_j >$

$$\varphi_9 = \bigwedge_i \bigwedge_j (I(j, i) \Rightarrow S(q_j', i + 1))$$

If instruction j was performed at time i, then state at time $i + 1$ must be q_j'.

Variables:

$\left\langle q_j, b_j, q_j', b_j', d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_{10}: The character written on tape that triggered an instruction, is the correct one

$$\varphi_{10} = \bigwedge_i \bigwedge_h \bigwedge_j [(I(j, i) \bigwedge H(h, i)) \Rightarrow T(b_j, h, i)]$$

If instruction j was executed at time i and head was at position h, then cell h has the symbol needed to issue instruction j is written under the head location on the tape.

Variables:

$\left\langle q_j, b_j, q_j', b_j', d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_{11}: The correct symbol was written to the tape at time i

$$\varphi_{11} = \bigwedge_i \bigwedge_j \bigwedge_h [(I(j, i) \wedge H(h, i)) \Rightarrow T(b'_j, h, i+1)]$$

If instruction j was executed time i with head at h, then at next time step symbol b'_j was written in position h

Variables:

$\left\langle q_j, b_j, q'_j, b'_j, d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

φ_{12}: Head was moved in the right direction at time i

$$\varphi_{12} = \bigwedge_i \bigwedge_j \bigwedge_h [(I(j, i) \wedge H(h, i)) \Rightarrow H(h + d_j, i + 1)]$$

The head is moved properly according to instr j.

Variables:

$\left\langle q_j, b_j, q'_j, b'_j, d_j \right\rangle$: jth instruction of M

$I(j, i)$: Instruction j was issued at time i.

$H(h, i)$: The head is at location h at time i.

$T(c, h, i)$: The tape at location h at time i stored the character c.

THE END

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(for now)