24.3.2
Problems related to graph coloring

Register allocation during compilation

1. When a compiler generates the assembly/VM code it needs to allocation registers to values being handled.
2. Need to make sure registers are not in conflict.
3. Build a conflict graph.
4. Color the conflict graph.
5. Every color is a register.
6. If not enough registers, then use memory/stack to store values.
7. CISC v.s. RISC.

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Graph Coloring and Register Allocation

Register Allocation

Assign variables to (at most) k registers such that variables needed at the same time are not assigned to the same register

Interference Graph

Vertices are variables, and there is an edge between two vertices, if the two variables are "live" at the same time.

Observations

- [Chaitin] Register allocation problem is equivalent to coloring the interference graph with k colors
- Moreover, **3-COLOR** $\leq_P$ **k-Register Allocation**, for any $k \geq 3$

Class Room Scheduling

1. Given n classes and their meeting times, are k rooms sufficient?
2. Reduce to Graph k-Coloring problem
3. Create graph G
   - a node v_i for each class i
   - an edge between v_i and v_j if classes i and j conflict
4. Exercise: G is k-colorable $\iff$ k rooms are sufficient.

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Frequency Assignments in Cellular Networks

1. Cellular telephone systems that use Frequency Division Multiple Access (FDMA) (example: GSM in Europe and Asia and AT&T in USA)
   - Breakup a frequency range $[a, b]$ into disjoint bands of frequencies $[a_0, b_0], [a_1, b_1], \ldots, [a_k, b_k]$
   - Each cell phone tower (simplifying) gets one band
   - Constraint: nearby towers cannot be assigned same band, otherwise signals will interference
2. Problem: given k bands and some region with n towers, is there a way to assign the bands to avoid interference?
3. Can reduce to k-coloring by creating interference/conflict graph on towers.

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THE END

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(for now)